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CALCULATING THE GAS CONCENTRATION IN APPARATUS EQUIPPED

WITH AGITATORS

V. M. Barabash, L. N. Braginskii,
and G. V. Gorbacheva

A method is proposed for calculating the mean gas concentration and its distribution along the height of an apparatus equipped with an agitator. An equation is derived theoretically for calculating the effective velocity of gas held on the surface of a gas—liquid layer.

One of the basic characteristics of a gas—liquid layer in apparatus equipped with agitators that influence the extent of the phase contact surface and mass-transfer intensity is its gas concentration.

According to the experimental results, the mean gas concentration increases as specific power and effective gas velocity increase [1-8]. A comparison of several empirical dependences showed [9] that their application led to divergent results which largely limits the possible use of the available correlations. This calls for urgently deriving generalized methods of calculating the gas concentration.

Apparatus equipped with agitators for mixing gas—liquid systems are also equipped with baffle plates and operate under turbulent conditions. Taking into consideration the results of studying the suspension mixing processes in such apparatus [10] and gas—liquid systems in bubblers [11], a single-parameter diffusion model could be used for describing the vertical gas bubble transport. This assumes the absence of a gas-phase concentration gradient along the radius of the apparatus and the presence of uneven gas concentration only in the axial direction:

$$v_b \frac{d\varphi}{dh} - D_t \frac{d^2\varphi}{dh^2} = 0. \tag{1}$$

For low gas concentrations, the buoyancy of bubbles ignoring restraints is [12]

$$v_b = \sqrt{2\sigma/[d_{bub}(\rho-\rho')] + \tfrac{1}{2}g d_{bub}(1-\rho'/\rho)}. \tag{2}$$

and the bubble diameter in the main section of the apparatus is [1]

$$d_{bub} = 4.15(\sigma/\rho)^{0.6}(\varepsilon_*^{av})^{-0.4} q_{av}^{0.5} + 0.0009. \tag{3}$$

The mean coefficient of eddy diffusion D_t in an apparatus equipped with baffle plates is determined using the equation of [9]

$$D_t = 0.435 n d_a D \left(\frac{\varepsilon_a^2 a}{R_d^3 v} \right)^{1/4}, \tag{4}$$

where $\gamma = 4H/R + 1$; $R_d = D/d_a$; $R = D/2$.

For the given conditions

$$v_b \varphi - D_t \frac{d\varphi}{dh} = w'; \tag{5}$$

$$\varphi = \varphi_s \quad \text{at} \quad h = H_{g-liq} \tag{6}$$
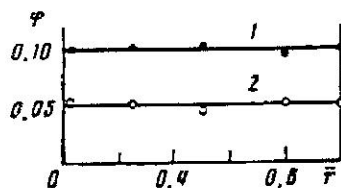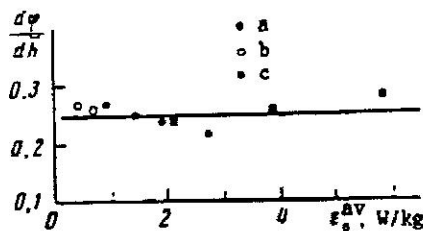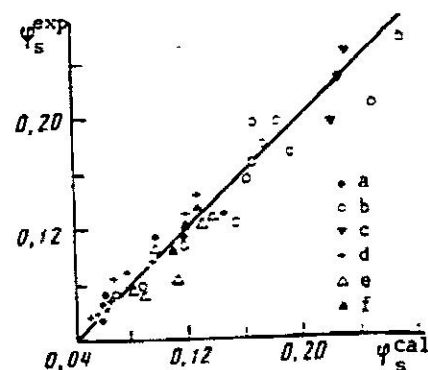
Fig. 1   Fig. 2   Fig. 3

Fig. 1. Gas concentration distribution along the radius of the apparatus ($D = 0.5$ m, $d_a =$ 0.15 m, $\xi_a = 8.4$, $H = 0.5$ m, $G_g = 5.7$ m^3/h, and $n = 10$ sec^{-1}); $\bar{h} = 1.0$ (1) and 0.5 (2).

Fig. 2. Dependence of gas concentration gradient in the near-surface zone on the mean energy dissipation: a and b) open turbine-type agitator: $D = 0.5$ m (a) and 0.24 (b); and c) agitator with three blades, $D = 0.24$ m.

Fig. 3. Comparison of the calculated and experimental values of near-surface gas concentration; a-c) open turbine-type agitator: a) $D = 0.5$ m and $d_a = 0.15$ m, b) 0.24 and 0.1, and c) 0.24 and 0.125; and d) $D = 0.24$ m and $d_a = 0.08$ m, rotary agitator; e) $D = 0.24$ m and $d_a =$ 0.125 m, agitator with two blades; and f) $D = 0.24$ m and $d_a = 0.125$ m, agitator with three blades.

a solution to Eq. (1) will have the form

$$\varphi(h) = \left(\varphi_s - \frac{w'}{v_b}\right) \exp\left(\frac{v_b(h - H_{g\text{-liq}})}{D_c}\right) + \frac{w'}{v_b}. \tag{7}$$

By integrating (7), we derive an equation for the mean gas concentration in the apparatus:

$$\varphi_{av} = \left(\varphi_s - \frac{w'}{v_b}\right)\left(1 - \exp\left(-\frac{v_b H_{q\text{-liq}}}{D_t}\right)\right)\frac{D_t}{v_b H_{q\text{-liq}}} + \frac{w'}{v_b}. \tag{8}$$

It may be seen from (7) and (8) that the surface gas concentration φ_s determined by the gas bubble transport pattern close to the liquid surface, together with the structural features of the apparatus and the agitator, phase properties, and also the gas flow through the bubbler, exerts much influence on the distribution of the dispersed phase in the apparatus and the mean concentration.

Transport in the near-surface zone of the gas–liquid layer should satisfy the gas balance

$$v_b \varphi_s - \frac{D_t^{loc}}{H_{q\text{-liq}}} \frac{d\varphi}{dh}\bigg|_{\bar{h} \to 1} = w' + w_s'. \tag{9}$$

The local coefficient of eddy diffusion D_t^{loc}, according to [13], could be determined from the local energy dissipation in the near-surface zone ε_0^{loc}:

$$D_t^{loc} = (\varepsilon_0^{loc})^{1/3}(\alpha R)^{4/3}. \tag{10}$$

The value ε_0^{loc} is determined as the sum of the dissipation of energies introduced by the agitator $(\varepsilon_0^{loc})_a$ and by the gas when feeding it through the bubbler $(\varepsilon_0^{loc})_g$. Considering that $(\varepsilon_0^{loc})_a \sim 10\%$ of the mean dissipation of energy ε_0^{av} [14] introduced into an apparatus equipped with an agitator and the energy introduced by the gas is dispersed practically uniformly throughout the apparatus, ε_0^{loc} could be expressed as

$$\varepsilon_0^{loc} \approx 0.1\varepsilon_0^{av} + w'g. \tag{11}$$

The intensity of gas flow into the apparatus from the surface is characterized by the effective gas velocity w'_s. The phenomenon of gas holdup on the liquid surface was studied in [7, 15-18]. It was shown [15] that gas holdup occurs only at an agitator rps exceeding
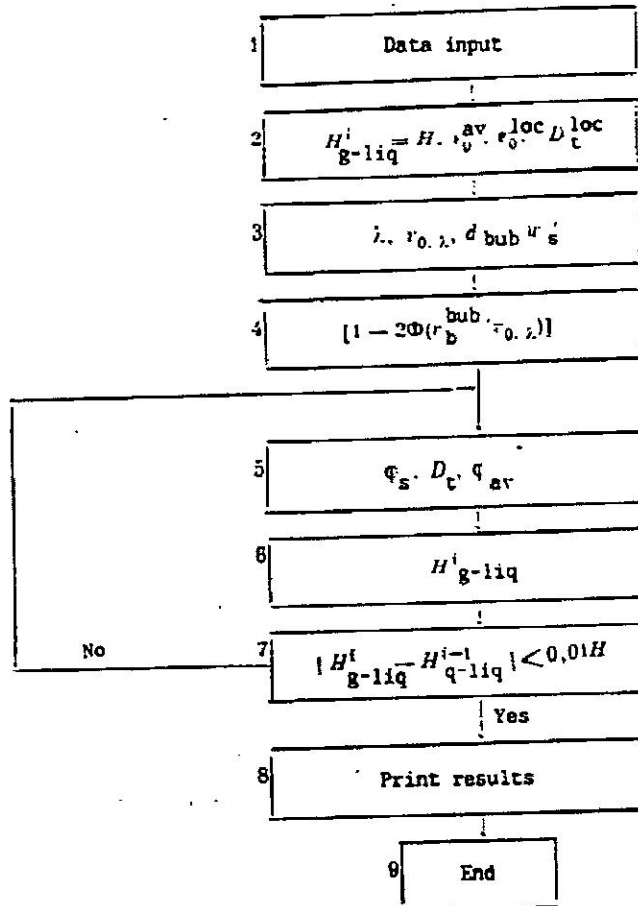
1	Data input

| 2 | $H^i_{g-liq} = H \cdot r^{av}_v \cdot r^{loc}_0 \cdot D^{loc}_t$ |

| 3 | $\lambda, r_{0,\lambda}, d_{bub}, w'_s$ |

| 4 | $[1 - 2\Phi(r^{bub}_b \cdot r_{0,\lambda})]$ |

| 5 | $q_s \cdot D_t, q_{av}$ |

| 6 | H^i_{g-liq} |

| 7 | $|H^i_{g-liq} - H^{i-1}_{q-liq}| < 0.01H$ |

Fig. 4. Iterative procedure for calculating the gas concentration in an apparatus equipped with an agitator.

$$\frac{\mu n^* d_a^2}{D\sigma}\left(\frac{\rho\sigma}{g\mu}\right)^{1/4} = 2\left(\frac{H-h_a}{D}\right)^{1/2}. \tag{12}$$

was derived for open turbine-type agitators at $0.1 \le [(H - h_a)/D] \le (0.25 \pm 1.75\, d_a/D)$. The dependence of n^* on the distance h_a between the agitator and the liquid surface has been derived in [16]. From the test results in a water—air system in apparatus of volume 4 to 100 m³ with one or more rows of turbine-type agitators, the following equation was derived [16]

$$\text{Fr}\,(d_a/h_a) = 0.36 \tag{13}$$

where Fr $= (n^*)^2 d_a/g$ is the Froude number.

The experimental dependences for calculating the gas holdup on the surface including those derived in [17 and 18] are qualitative while some quantitative evaluations led to contradictory results.

For a more accurate evaluation of the factors influencing the gas withdrawal from the surface, the mechanism of this phenomenon is analyzed. According to observations, small turbulent eddy formations, cavities, arise periodically on the surface as a result of its deformation and are whipped up causing increased gas concentration in the liquid layer.

At fairly small sizes of these cavities, i.e., under conditions when the effect of gravitational forces is insignificant, the magnitude of least surface deformations λ is determined from the equilibrium conditions of capillary and pulsating pressures:

$$\rho v^2_{s,\lambda}/2 = 2\sigma/\lambda, \tag{14}$$

hence

$$\lambda = 4\sigma/(\rho v_{0,\lambda}^2). \tag{15}$$

Having determined $v_{0,\lambda}^2$ using the "two-thirds law" of Kolmogorov–Obukhov [19 and 20]:

$$v_{0,\lambda} = (\varepsilon_0^{loc}\lambda)^{\frac{1}{3}} \tag{16}$$

we obtain

$$\lambda = 2.3(\sigma/\rho)^{0.6}(\varepsilon_0^{loc})^{-0.4}. \tag{17}$$

Assuming further that the frequency of surface deformations at a given point is equal to the frequency of velocity pulsations of magnitude λ:

$$v_{0,\lambda} = v_{0,\lambda}/\lambda = (\varepsilon_0^{loc})^{1/3}/\lambda^{2/3}, \tag{18}$$

for a unit surface with allowance for the local gas concentration φ_S, we have

$$v_{0,g} = v_{0,\lambda}(1-\varphi_g)/(\pi\lambda^2) = 0.32(\varepsilon_0^{loc})^{1/3}\lambda^{8/3}(1-\varphi_g). \tag{19}$$

Not all the pulsations of magnitude λ lead to the holdup of bubbles formed during the whipping up of the cavities but only those whose velocity exceeds the buoyancy of the bubbles v_b^{bub}. The proportion of such pulsations in the spectrum of pulsating velocities of magnitude λ assuming their normal distribution pattern [21] could be determined in the form

$$P = 1 - 2\Phi(v_b^{bub}/v_{0,\lambda}), \tag{20}$$

where $2\Phi(v_b^{bub}/v_{0,\lambda})$ is the probability that the pulsation rate $v_{0,\lambda}$ does not exceed the buoyancy of the bubbles formed on the surface in a normal distribution pattern [22].

From (19) and (20), we derive an equation for the frequency of bubble hold up from the surface

$$v_s = v_{0,g}P = \frac{v_{0,\lambda}}{\pi\lambda^2}(1-\varphi_g)\left[1-2\Phi\left(\frac{v_b^{bub}}{v_{0,\lambda}}\right)\right]. \tag{21}$$

From this, assuming the volume of the bubble holdup as roughly equal to the volume of the cavity, we obtain

$$w' = 0.88\left(\frac{\sigma}{\rho}\right)^{0.2}(\varepsilon_0^{loc})^{0.2}(1-\varphi_g)\left[1-2\Phi\left(\frac{v_b^{bub}}{v_{0,\lambda}}\right)\right]. \tag{22}$$

From (9), with allowance for (22), an equation could be derived for determining the gas concentration in the near-surface zone as

$$\varphi_g = \frac{w' + 0.88(\sigma/\rho)^{0.2}(\varepsilon_0^{loc})^{0.2}[1-2\Phi(v_b^{bub}/v_{0,\lambda})] + (D_t^{loc}/H_g \cdot 11q)(d\varphi/d\bar{h})|_{\bar{h}\to 1}}{v_b^{bub} + 0.88(\sigma/\rho)^{0.2}(\varepsilon_0^{loc})^{0.2}[1-2\Phi(v_b^{bub}/v_{0,\lambda})]} \tag{23}$$

Equations (4), (7), (8), (10), (11), and (23) help determine the gas concentration and its distribution along the height of the apparatus relative to the basic structural characteristics of the apparatus and the agitator, phase properties, and the gas flow through the bubbler. It should, however, be pointed out that, for using these equations, information is required on the proportionality coefficient α in (10) and the concentration gradient in the near-surface zone $(d\varphi/d\bar{h})|_{\bar{h}\to 1}$.

To test the assumptions made in designing the mathematical model and also to determine the values α and $(d\varphi/d\bar{h})|_{\bar{h}\to 1}$, gas concentration was measured in the near-surface zone as well as in the main body of the apparatus.

Tests were carried out in air-water system in apparatus of diameters 0.2, 0.25, and 0.5 m filled to a height of (0.7-3) D. The volume of the apparatus 15-100 liters. Agitation was by standard open turbine-type agitators with six blades ($h_{bl} = 0.2 d_a$), two blades ($h_{bl} = 0.1 d_a$), three inclined blades ($\alpha_a = 30°$ and $h_{bl} = 0.2 d_a$), and rotary agitators ($z_{bl} = 18$ and $h_{bl} = 0.11 d_a$) under developed turbulent conditions (Re > 10^4). The height of the agitator above the bottom of the apparatus was (0.5-1) d_a and the ratio of the diameters of the apparatus and agitator varied from 2 to 4.
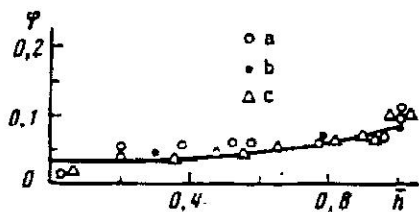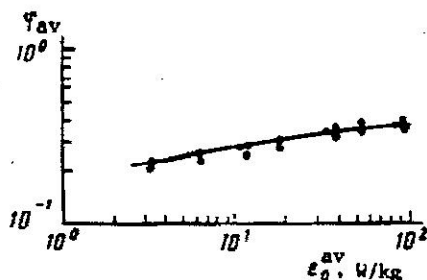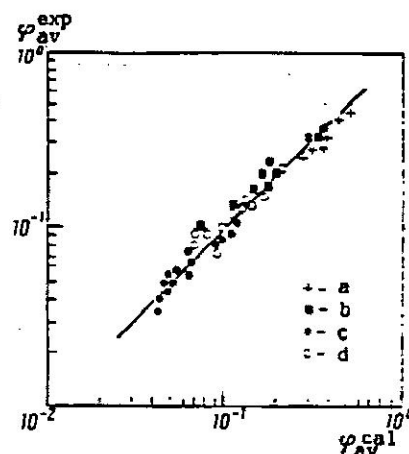
Fig. 5

Fig. 6

Fig. 7

Fig. 5. Gas concentration distribution along the height of the apparatus ($D = 0.5$ m, $H = 0.5$ m, $d_a = 0.15$ m, $n = 6$ sec^{-1}, $\xi_a = 8.4$, and $w' = 5.7 \times 10^{-3}$ m/sec); curve) theoretical; points) experimental; a) $r = 1$, b) 0, and c) 0.5.

Fig. 6. Dependence of mean gas concentration on the mean energy dissipation ($D = 0.25$ m, $d_a = 0.14$ m, open turbine-type agitator, $G_g = 8.3$ m^3/h, and $w' = 0.046$ m/sec): line) theoretical; and points) experimental [7].

Fig. 7. Comparison of calculated and experimental values of mean gas concentration: a) $D = 0.25$ m and $d_a = 0.14$ m, b) 0.25 and 0.098, c) 0.5 and 0.15, and d) 0.2 and 0.1; a and b) data of [7]; and c and d) data of the present authors.

Measurements were made using an "Impul's 05" device whose main element is a needle-type conductometric gauge registering and converting the changes of gas—liquid conductivity into electrical impulses whose duration is proportional to the residence of the electrode tip in the air bubble.

The experimental results of studying the gas concentration distribution in an apparatus equipped with an agitator justified the assumption of the absence of gas-phase concentration gradient in the radial direction (Fig. 1). The results of measuring the gas concentration along the height of the apparatus show that, in the immediate proximity of the surface, the gas concentration gradient could be regarded as constant at 0.25 with an accuracy adequate for engineering calculations (Fig. 2). The direct determination of $(d\varphi/d\bar{h})|_{\bar{h}=1}$ helped determine the exact proportionality coefficient α in (10). A comparison of the experimental values of gas concentration in the near-surface zone measured at a distance of 2-7 mm from the surface with the calculated values (Fig. 3) at $\alpha = 0.45$ suggests that the equation for gas holdup derived from the model under consideration agrees well with the results of measurements. The value of α is wholly reasonable as it corresponds not only with the results obtained for apparatus equipped with agitators [9] and bubblers [11] but also with the results of studying a much broader range of hydrodynamic problems [23] based on the hypothesis of the course of mixing advanced by Prandtl.

Equations (7) and (8) together with the values of $(d\varphi/d\bar{h})|_{\bar{h}=1}$ and α found help calculate the mean value of gas concentration and its distribution along the height of the apparatus. It should, however, be pointed out that, for calculating the gas concentration on the surface, its distribution along the height of the apparatus and its mean value for the height of the gas—liquid layer are not known initially. The calculation is, therefore, made by iteration for the value H_{g-liq}. The initial value of H_{g-liq} is assumed as equal to the height of the liquid bed H before feeding the gas into the apparatus and each subsequent approximation of the height of the gas—liquid layer is determined from the equation

$$H_{g-liq} = H/(1-\varphi_{av}).$$ (24)

The value φ_{av} is calculated from Eq. (9) using the characteristics of the preceding iteration. The scheme for calculations is shown in Fig. 4.

The gas concentration distribution along the height of the apparatus obtained by experimental measurements and also the theoretical dependence of φ on h constructed using Eqs. (4), (7), (8), (10), (11), and (23) are shown in Fig. 5. The dependence of mean gas concentration

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on $\epsilon_0{}^{av}$ is shown in Fig. 6. A comparison of the experimental and calculated values of mean gas concentration in apparatus of volume 15 to 100 liters is shown in Fig. 7. It may be seen from Figs. 6 and 7 that the theoretical dependences describe the experimental results quite well and could be recommended for use in the engineering practice.

NOTATION

D) Diameter of apparatus, m; D_t) coefficient of eddy diffusion, m^2/sec; d_a) diameter of agitator, m; C_g) gas flow, m^3/h; H) liquid height in the apparatus, m; H_{g-liq}) height of gas-liquid layer in the apparatus, m; h) current coordinate of height, m; h_a) height of agitator above the bottom of the apparatus, m; h_{bl}) height of blade, m; n) rps of agitator; v_b) rate of buoyancy of the bubble, m/sec; $v_{0,\lambda}$) pulsating velocity of magnitude λ, m/sec; w') effective gas velocity, m/sec; z_a) number of agitators on a shaft; z_{bl}) number of blades of agitator; α_a) angle of agitator blade inclination, deg; ϵ_0) rate of energy dissipation, W/kg; μ) fluidity of the continuous medium, Pa-sec; ζ_a) coefficient of resistance of agitator; ρ and ρ') densities of the liquid and gaseous phases, kg/m^3; σ) surface tension at the phase separation boundary, N/m; q, q_{av}, and q_s) gas concentration: current, mean, and on surface; $q_{av}{}^{exp}$ and $q_{av}{}^{cal}$) experimental and calculated values of mean gas concentration; $\bar{h} = h/H_{g-liq}$) relative coordinate of height; and $\bar{r} = r/R$) relative radius.

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PHENOMENOLOGICAL THEORY OF FAST MOTIONS OF A GRANULATED

MEDIUM BASED ON STATISTICAL MECHANICS METHODS

I. V. Shirko and V. A. Sakharov

UDC 539.215

On the basis of methods developed in the theory of Brownian motion, we have obtained a closed system of phenomenological equations describing "fast motions" of a granulated medium. Assuming that the viscous forces are small, we have shown that the distribution function of the velocities and displacements is a normal distribution. Using Prandl's concept of mixing path length we obtain the values of the viscosity and "thermal conductivity" coefficients, which proved to be dependent on the pressure and energy of random motion. The validity of the equations obtained is tested by comparing with the experimental ones in one-dimensional problems (Couette flow, flow in a vertical channel).

Beginning with the fundamental work of Coulomb and up to the beginning of the 1960's, the mechanics of granulated media has been developed as the science of limiting equilibrium. This is explained by the fact that for the planar deformed state and in the axially symmetric case, the problems of the mechanics of soils [1], like the theory of ideal plasticity, are statically definable. Therefore, in many cases the values of the critical loads and the stress distribution may be obtained without considering the equations for the velocities.

Despite the great diversity of approaches to constructing the velocity fields, up to the present time the most correct approach remains the flow theory in [2], based on application of the associated law to the Coulomb yield criterion. The major advantage of this theory is the fact that the characteristics of the stress and velocity fields in this case coincide, and consequently the regions found in limiting equilibrium may be defined unambiguously. The results of [3-5], in which the lines of discontinuity of the stresses and velocities are studied in detail and on this basis new problems with mixed boundary conditions are formulated and solved, have imparted a conclusive form to this division of mechanics. A systematic exposition of these questions is found in [6], and a review of the later studies in this direction is found in [7].

However, there are classes of flow of granulated media in which the observed velocity fields are found to be in substantial disagreement with the results of theories based on the assumptions outlined above. These flows are characterized by relatively high values of the velocity (on the order of meters per second) and usually arise in gravitational flows (movement of grain in silo elevators, ore material in ore chutes, fuel elements and catalyst granules in some nuclear and chemical reactor designs, etc.). It turns out that the behavior of materials for such types of flow is very similar to the behavior of a viscous liquid under analogous conditions.

Theoretical work in which these phenomena are studied have been formulated recently into an independent scientific direction called "the theory of fast motions of granulated media."

At the present time, the development of "viscous" effects for fast flows of absolutely dry granulated media is explained by the fact that for longitudinal "macroscopic motion" of shear flow, small displacements of the particles develop in the transverse direction, which transfer and thus cause the appearance of additional tangential stresses. Attempts at a quantitative description of such phenomena have been mainly undertaken in two directions: in the first case, familiar concepts from the kinetic theory of dense gases are drawn upon; in the second case, elements of the theory of turbulent motion of a liquid are used. Some papers by foreign authors and a review of studies in this direction are found in [8].