CHARACTERISTICS OF MACROSCALE TRANSFER DURING MIXING IN APPARATUS WITH REFLECTING PARTITIONS

L. N. Braginskii, Yu. V. Kokotov,
and L. S. Gordeev

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The authors propose a mathematical model of the process which simultaneously takes into account the meridional circulation of liquid, turbulent diffusion in the zones of ascending and descending streams and turbulent exchange between these zones.

The installation of reflecting partitions in apparatus with mixers is one of the commonest methods of intensifying mixing. Although a considerable number of papers have been devoted to the study of these apparatus, no single approach has yet been developed regarding the mechanism of mixing in them, and the method for calculating the concentration and temperature field is in most cases based on empirical dependences [1].

A large number of mathematical models have been proposed to describe the processes of macroscale transfer in apparatus with partitions. The analysis and comparison of these models with experimental results show that the process can be satisfactorily described in most cases by means of both diffusion and circulation models [2-5]. However, it is usually not possible to determine the parameters of these models on the basis of independently obtained hydrodynamic data. Even a stochastic model with a variable structure [6, 7], together with an independently determined circulation liquid flow rate includes one parameter – the number of cells – which is determined from the results of comparison of the model equations with the apparatus. The difficulties of mathematical modeling of apparatus with reflecting partitions are associated above all with the complex character of flow of the medium, since the stream in the apparatus is essentially nonstationary in nature, while the macroscale velocity pulsations are comparable in magnitude with the velocity of the averaged flow.

Existing data on the characteristics of the velocity field in these apparatus [6-8] indicate that macroscale transfer in apparatus of the type under consideration is determined simultaneously by meridional circulation and turbulent diffusion superimposed on it. Here, in contrast to apparatus without internal devices [9], the turbulent diffusion not only promotes longitudinal transfer in the circulation circuit, but also causes mixing in the transverse direction between the ascending and descending streams.

Thus, where the mixer is located near the bottom [10], the apparatus volume can be represented as consisting of two concentric zones – the ascending and the descending streams, mixing in each of which is the result of turbulent diffusion – and exchange takes place between them through circulation and turbulent diffusion (Fig. 1).

Considering that the cross sections of the zones are equal [7], the process of mixing is described in such a model by the system of equations [11, 12]

$$\frac{\partial u}{\partial \tau}-\frac{1}{\mathrm{Pe}}\frac{\partial^2 u}{\partial x^2}+\frac{\partial v}{\partial x}=0;$$
$$\frac{\partial v}{\partial \tau}-\frac{1}{\mathrm{Pe}}\frac{\partial^2 v}{\partial x^2}+\frac{\partial u}{\partial x}+2Av=0, \tag{1}$$

where

$$u=(C_1+C_2)/2;\quad v=(C_1-C_2)/2;\quad A=16\sqrt{2}(H/D)^2\mathrm{Pe}^{-1}. \tag{2}$$

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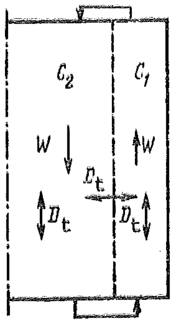

Fig. 1. Diagram of the diffusion–circulation model.

The dimensionless parameters A and Pe entering into system (1) are determined by two characteristics of transfer, namely axial flow velocity W and turbulent diffusion coefficient D_t, which depend on the design characteristics of the apparatus.

The axial flow velocity can be determined from the magnitude of the circulation flow rate of liquid. Expressing the circulation flow rate in accordance with [13, 14] as

$$q=1.8k_q n d_m^3,$$

we have

$$W=14.4k_q n d_m^3/(\pi D^2). \tag{3}$$

To determine turbulent diffusion coefficient, we use the Kolmogorov–Obukhov hypothesis [15]

$$D_t\simeq\lambda v_\lambda'\simeq\lambda^{4/3}\varepsilon_0^{1/3}, \tag{4}$$

where ε_0 is the average value of energy dissipation in unit mass, W/kg:

$$\varepsilon_0=N/(\rho V_{ap})=k_N n^3 d_m^5/V_{ap}. \tag{5}$$

In (4) λ can be regarded as a function of the characteristic geometric dimension determining the scale of the largest eddies.

In apparatus with reflecting partitions an intensive eddy formation takes place in the zone of flow around the mixer blades and in that around the partitions. The calculated evaluations based on the data of [13] show that the velocity of flow around the blades exceeds that around the partitions by a factor of 3 to 5, and since the linear dimensions of the blades, h_b and those of the partitions, S_h, are close, the most intensive eddying should take place during the flow around the blades. This conclusion is in agreement with experimental data [16], which indicate that the macroscale of turbulence in apparatus with partitions is comparable with blade height, and enables us to assume

$$\lambda\sim h_b$$

or

$$\lambda=\varkappa h_b, \tag{6}$$

where $\varkappa$ is the coefficient of proportionality of the order of unity.

Taking into account (4)-(6) we obtain for the turbulent diffusion coefficient

$$D_t\simeq n(\varkappa h_b)^{4/3}[4k_N d_m^5/(\pi D^2 H)]^{1/3}. \tag{7}$$

Expressions (3) and (7) link the hydrodynamic parameters of the mathematical model to design characteristics of the apparatus and mixer; they make it possible to close system (1) and use it for calculating the transfer process.

Experimental verification of the adequacy of the model was carried out using as an example the equalization of the concentration of a tracer which was introduced in a pulsed manner into the upper part of the central zone of the apparatus. The initial conditions corresponding to this case are

$$v|_{\tau=0}=0; \tag{8}$$

$$u|_{\tau=0} = \delta^{+}(x), \tag{9}$$

where $\delta^{+}(x)$ is the one-sided δ-function permitting in the interval [0, 1) the Fourier expansion

$$\delta_{+}(x) = 1 + 2\sum_{i=1}^{\infty} \cos \pi i x,$$

and the boundary conditions

$$v\Big|_{\substack{x=0\\x=1}} = 0, \tag{10}$$

$$\frac{\partial u}{\partial x}\Big|_{\substack{x=0\\x=1}} = 0. \tag{11}$$

Conditions (8), (10), and (11) reflect the absence of diffusion transfer through the outer cross sections of the apparatus, and condition (9), the pulsed nature of introduction of the tracer.

An approximate solution of this problem was found in [12] which is valid for the condition $A \gg 1$ characteristic of bubble apparatus. We seek an accurate solution valid for any A in the form

$$u = \sum_{k=0}^{\infty} A_k(\tau) \cos \pi k x; \quad v = -\sum_{k=0}^{\infty} B_k(\tau) \sin \pi k x. \tag{12}$$

Substituting (12) into (1), we obtain the system

$$\frac{dA_k}{d\tau} = -\frac{\pi^2 k^2}{\mathrm{Pe}} A_k - \pi k B_k; \quad \frac{dB_k}{d\tau} = -2AB_k + \pi k A_k - \frac{\pi^2 k^2}{\mathrm{Pe}} B_k, \tag{13}$$

the solution of which with the initial conditions

$$A_0|_{\tau=0} = 1; \quad A_k|_{\tau=0} = 2 \text{ for } k = 1, 2, \ldots, \infty; \quad B_k|_{\tau=0} = 0, \tag{14}$$

obtained by substitution of (13) into (8) and (9) takes the form

$$B_0 = 0, \quad A_0 = 1, \tag{15}$$

$$A_k = \begin{cases} 2\left(\mathrm{ch}\sqrt{A^2 - \pi^2 k^2}\,\tau + \dfrac{A\,\mathrm{sh}\sqrt{A^2 - \pi^2 k^2}\,\tau}{\sqrt{A^2 - \pi^2 k^2}}\right) \exp\left[-(A + \pi^2 k^2/\mathrm{Pe})\,\tau\right] & \text{for } k < \dfrac{A}{\pi}; \\ 2\left(\cos\sqrt{\pi^2 k^2 - A^2}\,\tau + \dfrac{A \sin\sqrt{\pi^2 k^2 - A^2}\,\tau}{\sqrt{\pi^2 k^2 - A^2}}\right) \exp\left[-(A + \pi^2 k^2/\mathrm{Pe})\,\tau\right] & \text{for } k \geqslant \dfrac{A}{\pi}; \end{cases} \tag{16}$$

$$B_k = \begin{cases} \dfrac{2\pi k\,\mathrm{sh}\sqrt{A^2 - \pi^2 k^2}\,\tau}{\sqrt{A^2 - \pi^2 k^2}} \exp\left[-(A + \pi^2 k^2/\mathrm{Pe})\,\tau\right] & \text{for } k < \dfrac{A}{\pi}; \\ \dfrac{2\pi k \sin\sqrt{\pi^2 k^2 - A^2}\,\tau}{\sqrt{\pi^2 k^2 - A^2}} \exp\left[-(A + \pi^2 k^2/\mathrm{Pe})\,\tau\right] & \text{for } k \geqslant \dfrac{A}{\pi}; \end{cases}$$

$k = 1, 2, \ldots, \infty.$

Substituting expressions (15) and (16) into (13), we find for the average dimensionless concentration u in the lower cross section x = 1 the relation

$$u = 1 + 2\sum_{k=1}^{n_1} (-1)^k \left[\mathrm{ch}\sqrt{A^2 - \pi^2 k^2}\,\tau + \frac{A\,\mathrm{sh}\sqrt{A^2 - \pi^2 k^2}\,\tau}{\sqrt{A^2 - \pi^2 k^2}}\right] \times$$

$$\times \exp\left[-(A + \pi^2 k^2/\mathrm{Pe})\,\tau\right] + 2\sum_{k=n_1+1}^{\infty} (-1)^k \left[\cos\sqrt{\pi^2 k^2 - A^2}\,\tau + \frac{A \sin\sqrt{\pi^2 k^2 - A^2}\,\tau}{\sqrt{\pi^2 k^2 - A^2}}\right] \exp\left[-(A + \pi^2 k^2/\mathrm{Pe})\,\tau\right], \tag{17}$$

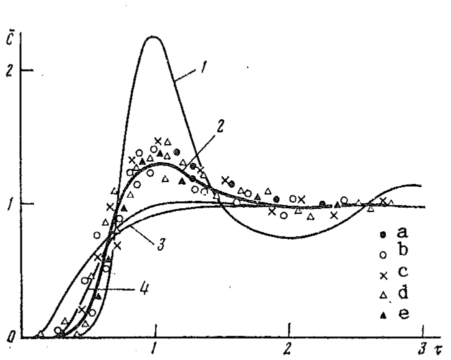

Fig. 2

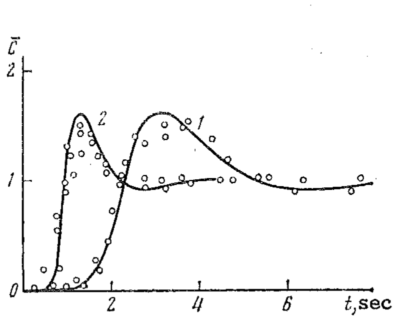

Fig. 3

Fig. 2. Dimensionless curves of equalization of concentration in a vessel with an open turbine mixer.

Curves	$\varkappa$	Pe	Points	H = D, m	d_m, m	n, sec^{-1}
1	0.4	36.1	a	1.5	0.5	2.8
2	0.7	36.1	b	0.16	0.075	2.2
3	2.0	4.23	c	0.16	0.075	4.17
4	1.0	10.67	d	0.16	0.05	5.0
			e	0.16	0.05	17.5

Fig. 3. Curves of equalization of concentration in a vessel with a two-blade mixer (d_m = 0.075 m, H = D = 0.16 m): 1) n = 5.0 sec^{-1}; 2) 12.0.

where η_1 is the whole part of the number A/π.

This expression containing two dimensionless parameters A and Pe for a fixed ratio of the height to the diameter of the apparatus, taking into account (9), can be represented in a form containing only one parameter Pe, which reflects in the given case the relationship between circulation intensity and diffusion transfer in the zones.

In the case of H/D = 1, Fig. 2 gives the dependence $u(\tau)$ for different values of Pe. It can be seen from the figure that as Pe increases, the circulation properties of the model emerge increasingly clearly.

The dimensionless experimetnal curves of concentration equalization are also plotted on the graph. Experiments were conducted in apparatus with diameters of 0.16 and 1.5 m having six-bladed turbine mixers [17] (h_b = 0.2 d_m). The method of measurement was that described in [9].

The results of comparison of the calculation and experimental results show that for values of $\varkappa$ from 0.55 to 0.8 the proposed model satisfactorily describes the qualitative character of the curves. Here the best quantitative agreement is obtained for values of $\varkappa$ = 0.7 (Fig. 2). In the calculations we took k_q = 0.5 and k_N = 5 [13, 14, 17] determined from direct measurements.

Figure 3 gives the similar calculated (for $\varkappa$ = 0.7) and experimental curves for two-blade mixers (h_b = 0.1 d_m, k_N = 0.6 [17], k_q = 0.24*).

The results of comparison, shown in Figs. 2 and 3, can be considered to be a confirmation of the adequacy of the model, including the equations for determining its hydrodynamic parameters during description of the process of macrotransfer in apparatus with turbine and blade mixers.

(2) In the proposed model, the characteristic scale of turbulence is linked to the height of the mixer blade ~~(Eq. (6))~~. This result differs from ~~the~~ other data ~~in [4]~~, where a single-parameter diffusion model was used to evaluate the inhomogeneity of distribution of the solid phase, and the scale of turbulence in the calculation of the turbulent diffusion coefficient was found to be 0.45 times the apparatus radius. At the same time, the results given

*The value k_q = 0.24 for a mixer of the given type was found from the value of the time corresponding to the maximum on the experimental curve.

in another study

~~in [4]~~ indicate that the single-parameter diffusion model predicts with sufficient accuracy both the nature of distribution of the solid phase along the height and the absolute concentration values.

In order to clear up the contradiction between the results of [4] and the present study, we solve the equations of the diffusion–circulation model, adding to system (1) terms which take into account the particle deposition rate:

$$\frac{1}{\mathrm{Pe}}\frac{\partial^2 u}{\partial x^2}-\frac{\partial v}{\partial x}-\alpha\frac{\partial u}{\partial x}=0; \tag{18}$$

$$\frac{1}{\mathrm{Pe}}\frac{\partial^2 v}{\partial x^2}-\frac{\partial u}{\partial x}-\alpha\frac{\partial v}{\partial x}-2Av=0, \tag{19}$$

where $\alpha = W_{dr}/W$ (W_{dr} is the rate of deposition of solid particles, m/sec; u and v are the half-sum and half-difference of solid-phase concentrations in the central and peripheral zones.

Considering that in apparatus with reflecting partitions, as shown above, $\mathrm{Pe} \gg 1$ ($\mathrm{Pe} \approx 20$), and evaluating the orders of the terms in Eq. (19), we obtain for the common case arising in practice $\alpha \ll 1$:

$$\frac{\partial u}{\partial x}\simeq -2Av, \quad \text{or} \quad v\simeq -\frac{1}{2A}\frac{\partial u}{\partial x}. \tag{20}$$

Substituting (20) into (18), we have

$$\left(\frac{1}{\mathrm{Pe}}+\frac{1}{2A}\right)\frac{\partial^2 u}{\partial x^2}-\alpha\frac{\partial u}{\partial x}=0, \tag{21}$$

or, returning to the dimensional variables:

$$D_{\mathrm{T}}\left(1+\frac{\mathrm{Pe}}{2A}\right)\frac{\partial^2 u}{\partial X^2}-W_{\mathrm{dr}}\frac{\partial u}{\partial X}=0. \tag{22}$$

Relation (22) is the equation of the single-parameter diffusion model with an effective diffusion coefficient $D_{ef} = (1 + \mathrm{Pe}/(2A))D_t$, which gives for an apparatus with a turbine mixer (where H/D = 1) $D_{ef} \simeq 7.4D_t$.

Taking into consideratin that $D_t \sim \lambda^{4/3}$, we obtain

$$\lambda_{ef}\simeq 7.4^{3/4}\lambda=4.5\lambda, \tag{23}$$

where λ_{ef} is the apparent value of λ if one uses the single-parameter diffusion model which disregards the phenomenon of liquid circulation.

In the case of $D/d_m = 3$, relation (23) taking into account (16) gives

$$\lambda_{ef}=0.21D=0.42R. \tag{24}$$

This result agrees satisfactorily with the data of [4], where $\lambda \simeq 0.45R$.

Thus, the calculation of apparatus with reflecting partitions which are intended for mixing of suspensions can be performed on the basis of the single-parameter diffusion model using the value of D_{ef} determined by Eqs. (4), (6) and (23).

NOTATION

$C_{1,2}$ is the average dimensionless concentrations over the cross section of the peripheral and central zones; D is apparatus diameter, m; d_m is mixer diameter, m; H is apparatus height, m; h_b is mixer blade height, m; k_q is flow rate coefficient; k_N is power factor; n is mixer rotation velocity, sec^{-1}; t is time, sec; $v\lambda'$ is characteristic velocity of turbulent pulsations of scale λ, m/sec; $x = X/H$ is the dimensionless coordinate; X is the vertical coordinate, m; $\tau = tW/H$ is dimensionless time; $\mathrm{Pe} = HW/D_t$ is Peclet number.

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