

THE INFLUENCE OF VISCOSITY ON THE PERIPHERAL VELOCITY OF A LIQUID IN AN APPARATUS WITH A STIRRER

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The authors have experimentally studied the influence of viscosity (from 0.001 to 8.5 N·sec/m²) on the distribution of the peripheral component of the velocity of a liquid in apparatus (without internal structures) with stirrers of various types. They determine values of the coefficient of resistance to flow of liquid in apparatus with stirrers, and derive equations linking the peripheral velocity to the power consumption of stirring, the properties of the medium, and the characteristics of the apparatus.

Braginskii [1] analyzed the radial distribution of the peripheral component of the velocity of a liquid in a stirring apparatus, and obtained approximate equations relating the peripheral velocity profile to the construction of the stirrer and apparatus. A more exact solution of certain practical problems would also involve the physical properties of the medium being stirred. In particular, in certain conditions the peripheral velocity distribution of the stirred liquid is considerably affected by its viscosity [2, 3]. However, the data in the papers cited do not permit a quantitative estimate of the influence of viscosity on peripheral flow during mixing. We have therefore measured the peripheral velocities in liquids of various viscosities (water, glycerin, and aqueous solutions of glycerin and syrup) with the aim of obtaining quantitative relations for the influence of viscosity on the flow characteristics during operation of the commonest types of stirrers.

The peripheral velocity measurements were made with the aid of the apparatus shown diagrammatically in Fig. 1. A circular cylinder 1, 8 mm in diameter and 100 mm in length, or alternatively a flat plate 20 × 25 mm in size, was attached by blade 2 and steel rod 3 to spindle 4, which rotated freely in its bearings. Scale and slider 7 permitted smooth radial adjustment of the body. Using the action of forces due to motion of the stirred liquid, body 1 deviated from the vertical in the direction of flow. With the aid of screw 8 it was returned to a strictly vertical position, and from the tension in spring 9 the moment acting on it was measured, and hence the force due to the flow. To avoid drift of the zero of scale 5 relative to the vertical, due to unavoidable tolerances in the scale and slider, the scale was mounted on a bearing and provided with plumb 6 and bob weights 10. The spring was calibrated with the aid of weights, for which we used pulley 11.

Before making the measurements, we calibrated the measuring bodies on a test rig similar to that described in [4]. The main component of this rig was a rotating finned cylindrical vessel, 0.4 m in diameter and 0.35 m in depth. Calibration was performed in liquids of various viscosities (from 0.001 to 8.5 N·sec/m²); the rate of flow past the measuring body was varied by altering the rate of rotation of the vessel and by shifting the device radially. We thus plotted calibration curves $\zeta = \varphi(\text{Re}_T)$, in the range $0.3 \leq \text{Re}_T \leq 20,000$ (Fig. 2), which we used in processing the results of our peripheral velocity measurements. To estimate possible distortion in the velocity distribution due to the measurement body, in the calibration test rig we carried out a series of experiments to measure the heat emission from the circular cylinder. For this purpose we made a cylindrical heating element of cuprite, within which was a winding heated by

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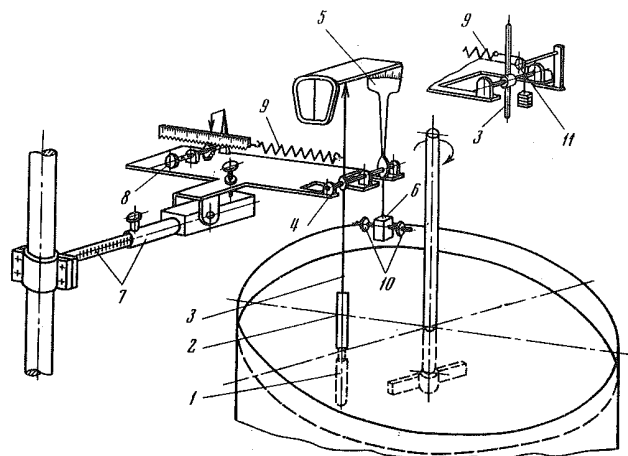


Fig. 1. Diagram of apparatus for measurement of peripheral velocity of liquid.

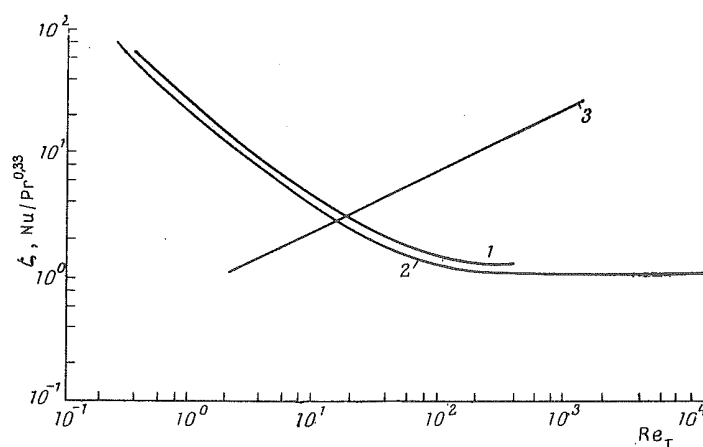


Fig. 2. Coefficient of resistance of measurement bodies, and heat emission of circular cylinder. 1) Resistance of plate; 2) resistance of cylinder; 3) heat emission of cylinder.

an electric current. Into the wall of the element were sealed two thermocouples. The heating element was 20 mm in diameter and 100 mm long. From measurements of the heat emission we found the relation $Nu/Pr^{1/3} = \varphi(Re_T)$; the velocity of the liquid flowing past the element was calculated as in the calibration of the measurement apparatus, starting from the rate of rotation of the vessel and the radial coordinate of the element. The experimental points obtained in these experiments lay along the well-known curve found by Gil'pert [5] for heat emission by a circular cylinder in a current. This comparison may be considered as evidence that the true velocity was practically the same as the calculated velocity, i. e., that the heating element does not cause appreciable distortion in the current. Obviously, measurement bodies with lower resistances will also not exert appreciable influence on the peripheral flow in the calibration vessel or in the apparatus with the stirrers. The results of experiments in the calibration rig also showed that the resistance of the blade 2 immersed in the liquid was small in comparison with the resistance of the measurement bodies, and thus the apparatus was suitable for measuring velocities at various depths below the surface.

The peripheral velocities were measured in apparatus with diameters of 0.3, 0.4, and 1 m with stirring by open and closed turbine, paddle, and propeller stirrers, between 50 and 300 mm in diameter, rotating at rates between 40 and 600 rpm. The liquid level height in the apparatus was equal to the

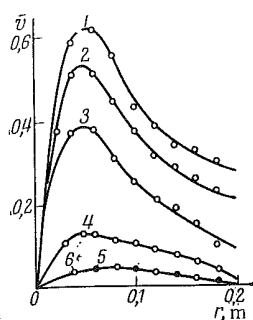


Fig. 3. Influence of viscosity on peripheral velocity profile. Open turbine stirrer 0.15 m in diameter, in apparatus 0.4 m in diameter.

Curve numbers	Viscosity, $N \cdot \text{sec}/m^2$	Re_c
1	0,001	45000
2	0,035	1260
3	0,0875	550
4	0,406	129
5	1,715	29
6	8,2	5,9

velocity profile undergoes no appreciable changes as the viscosity increases. The values of the viscosity and Re_c , corresponding to changes in the nature of the influence of the viscosity on the peripheral velocity distribution, are different for stirrers of different types and also for different ratios of the stirrer and vessel dimensions.

A quantitative correlation of the results may be based on analysis of the influence of viscosity on the power consumed by the stirrer and on the resistance of the liquid in the apparatus to flow. Using the condition that the torque on the stirrer shaft is equal to the moment of the resistance arising on the walls and bottom of the apparatus, we write

$$M_t = \frac{N}{\omega} = 0.129 K_N \rho \omega^2 r_0^5 = M_w + M_b. \quad (1)$$

The moment of resistance at the cylindrical wall of the apparatus is

$$M_w = 2\pi \tau R^2 H, \quad (2)$$

where

$$\tau = c_f \rho v^2 / 2. \quad (3)$$

Since the peripheral velocity of the liquid varies along the radius of the apparatus, the value of c_f may vary with the selected characteristic value of the peripheral velocity. For example, as the characteristic peripheral velocity value, Mizushina et al. [6] chose the peripheral velocity of the liquid near the wall of the apparatus. However, this method of expressing τ is insufficiently rigorous, because the velocity "near the wall" is an arbitrary quantity, and the point at which it is measured is not specified. Evidently for apparatus with stirrers, as for the flow of liquid in channels, it would be sounder to use an expression for the tangential stress, including the mean (flow) velocity, which in our case can be defined on the

*For stirring high-viscosity liquids with laminar flow, one usually makes use of stirrers with small gaps between the walls and the blades — anchor-type, frame-type, strip-type, etc.

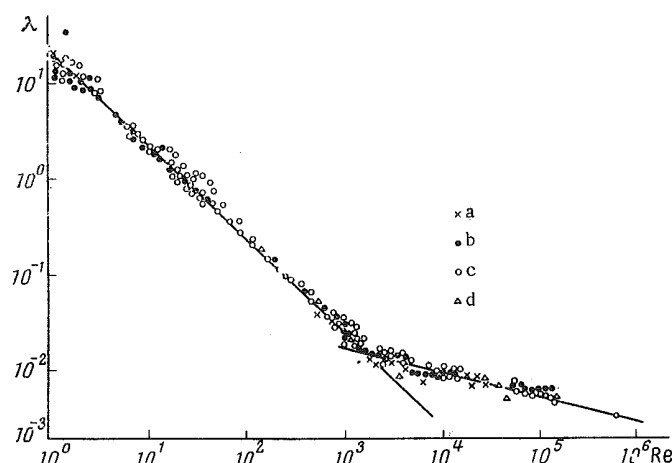


Fig. 4. Coefficient of resistance of walls and floor of apparatus versus Reynolds number. a) Closed turbine stirrer; b) open turbine stirrer; c) paddle stirrer; d) propeller stirrer.

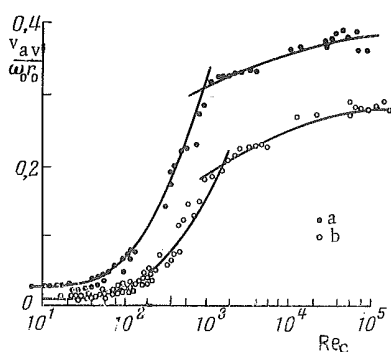


Fig. 5. Mean peripheral velocity versus centrifugal Reynolds number. a) Open turbine stirrer, $R/r_0 = 2.7$; b) paddle stirrer, $R/r_0 = 2.0$.

basis of the measurement results as

$$v_{av} = \frac{1}{R} \int_0^R v \, dr. \quad (4)$$

Hence

$$M_w = \pi c_f v_{av}^2 R^2 H \rho. \quad (5)$$

Strictly speaking, the moment of resistance of the bottom of the apparatus should be defined by integrating the expression

$$dM_b = \pi c_f \rho v^2 r^2 dr \quad (6)$$

from 0 to R . However, for simplicity in the ultimate expressions, it is more convenient to use the approximate relation

$$M_b = \pi c_f \rho v_{av}^2 R^3 / 4. \quad (7)$$

This does not lead to appreciable errors, because the area of the floor on which the flow velocity exceeds the mean value is small, and the tangential stresses which arise there are operating at small radii. Substituting these expressions for the moments into (1), we get

$$c_f = 0.165 K_N / \gamma \bar{R}^3 \bar{v}_{av}^2, \quad (8)$$

where $\gamma = 4H/R + 1$ for apparatus with free liquid surface, and $\gamma = 4H/R + 2$ for wholly filled apparatus.

Equation (8) was used to process the experimental data with the aim of finding the relation between the coefficient of resistance and the viscosity of the stirred liquid and its flow conditions. The values of K_N were found experimentally (by measuring the torque on the shaft by means of a spring dynamometer) or with the aid of graphs of K_N versus Re_c [7, 8]; and the values of \bar{v}_{av} were calculated from the experimental peripheral velocity profiles. From Fig. 4 we see that the values found for c_f for apparatus with the stirrers studied in this article all lie near a single curve. The steep part of this curve, characterizing laminar flow of the stirred liquid, corresponds to the theoretical relation for laminar flow in a plane channel [9]:

$$c_f = 24 / Re. \quad (9)$$

The flatter part of the curve, corresponding to turbulent stirring, is represented with satisfactory accuracy

by the expression

$$c_f = 0.095 \text{Re}^{-0.25}, \quad (10)$$

which is close to the well-known equation for resistance in smooth tubes [9]. The transition from laminar to turbulent stirring is observed at Reynolds numbers of about 1500. Substituting the expressions found for the coefficient of resistance, (9) and (10), into (8), after some transformations we obtain an equation relating the mean peripheral velocity in the apparatus to the conditions of stirring and the properties of the liquid:

laminar flow ($\text{Re} < 1.5 \cdot 10^3$):

$$\begin{aligned} (\text{Re} < 1.5 \cdot 10^3): \\ \bar{v}_{av} = 0.0107 K_N \text{Re}_C / \bar{R}^2 \gamma; \end{aligned} \quad (11)$$

turbulent flow ($1.5 \cdot 10^3 < \text{Re} < 10^6$):

$$\begin{aligned} (1.5 \cdot 10^3 < \text{Re} < 10^6) \\ \bar{v}_{av} = 1.47 [K_N \text{Re}_C^{0.25} / \bar{R}^{2.75} \gamma]^{0.57}. \end{aligned} \quad (12)$$

Figure 5 compares the results of a calculation of the mean velocity by means of Eqs. (11) and (12) with the experimental data for stirrers of two types – paddle and open turbine. The complex character of the influence of viscosity on the peripheral velocity, as revealed in this graph, can be explained if we compare the flow conditions of the liquid along the walls and the conditions for flow past the stirrer blades. * Since for stirrers with flat radial blades

$$dM_t = \frac{\zeta_B}{2} \rho (\omega_0 r - v)^2 n_B h_B r dr, \quad (13)$$

therefore

$$N = M_t \omega_0 = \frac{\rho_B n_B h_B \omega_0^3 r_0^4}{2} \int_0^1 \zeta_B (\bar{r} - \bar{v})^2 \bar{r} d\bar{r}. \quad (14)$$

Assuming that the main part of the moment is communicated to the liquid by the peripheral part of the blade, to a first approximation we can take the coefficient of resistance of the blade to be constant and can remove it from the integral sign:

$$K_N = \frac{n_B \zeta_B}{0.258} \frac{h_B}{r_0} \int_0^1 (\bar{r} - \bar{v})^2 \bar{r} d\bar{r}. \quad (15)$$

The condition of conservation of momentum (1), with the aid of (15) and Eqs. (6) and (7), can be written in the form

$$\frac{\zeta_B}{c_f} = 0.157 \frac{\gamma}{n_B} \frac{r_0}{h_B} \frac{\bar{v}_{av}^2}{\int_0^1 (\bar{r} - \bar{v})^2 \bar{r} d\bar{r}}. \quad (16)$$

Equation (16) shows that, for constant degree of influence of the viscosity of the medium on the coefficient of resistance of the blade and wall of the apparatus, the right-hand side of the equation characterizing the distribution of peripheral velocity must remain constant. As we see by comparing the curves $\zeta = \varphi(\text{Re}_T)$ and $c_f = \varphi(\text{Re})$ (Figs. 2 and 4), a constant degree of influence of the viscosity on the coefficients of resistance of the wall and flat plate will occur for laminar flow, which corresponds to the values $\text{Re}_C < 50$ in Fig. 5. For turbulent flow, the influences of viscosity on ζ and c_f also differ little; therefore for $\text{Re}_C > 1000$, corresponding to turbulent flow over the walls and blades, increase in viscosity has only a relatively slight effect on the velocity. According to the results of processing the experimental data, the region of intermediate values of the centrifugal Reynolds numbers (50–1000) is characterized by $\text{Re} = 10$ –1000 and $\text{Re}_B = 6$ –500. In these conditions the flow in the main part of the space in the apparatus is laminar (Fig. 4), and the flow over the plate corresponds to the transitional region between laminar and turbulent flow (Fig. 2). In this case the effects of the viscosity on ζ and c_f are quite different, and therefore the ratio ζ/c_f , and hence also the velocity distribution, varies markedly with the viscosity.

* It is assumed that when $\text{Re}_T = \text{Re}_B$ the conditions of flow past the stirrer blades are similar to the conditions of flow over the plate (Fig. 2).

Comparison of the results of calculations based on (11) and (12) with the experimental data (Fig. 5) reveals that these equations can be used to determine the mean peripheral velocities of liquid in apparatus with stirrers, with a degree of accuracy sufficient for solving engineering problems (the root-mean-square error of calculations based on Eq. (11) is 22%, that of calculations based on (12) is 10.4%).

The equation of mean velocity (12), together with expressions for the peripheral velocity found previously [1], enables us to calculate the radial distributions of the peripheral velocity in liquids of various viscosities under turbulent flow conditions. Figure 3 compares the velocity profiles calculated from these equations with the experimental data (curves 1 and 2).

NOTATION

a	is the coefficient of thermal diffusivity, m^2/sec ;
c_f	is the coefficient of resistance of walls and floor of apparatus;
d_M	is the diameter of stirrer, m ;
H	is the depth of filling of apparatus, m ;
h_B	is the width of stirrer blade, m ;
$K_N = (N/\rho n^3 d_M^5)$	is the dimensionless power;
l	is the characteristic dimension of body, m ;
M_t, M_w, M_b	are, respectively, torque and moments of resistance of walls and bottom, J ;
N	is the power, W ;
n	is the rate of rotation of stirrer, rev/sec ;
n_B	is the number of stirrer blades;
$r = r/r_0$	is the dimensionless radius;
R, r_0, r	are the radii of apparatus and stirrer, and radius coordinate, respectively, m ;
v	is the peripheral velocity, m/sec ;
v_{r_0}, v_{av}	are the peripheral velocity at radius r_0 , and mean peripheral velocity, respectively, m/sec ;
$\bar{v} = v/\omega r_0$	is the dimensionless velocity;
α	is the coefficient of heat emission, $\text{W/m}^2 \cdot \text{deg}$;
ξ_B, ξ	are the coefficients of resistance of blade and body;
λ	is the coefficient of thermal conductivity, $\text{W/m} \cdot \text{deg}$;
ν	is the coefficient of kinematic viscosity, m^2/sec ;
ρ	is the density, kg/m^3 ;
ω	is the angular velocity of stirrer, rad/sec ;
$Nu = \alpha l/\lambda$	is the Nusselt number;
$Pr = a/\nu$	is the Prandtl number;
$Re = v_{av} R/\nu$	is the Reynolds number;
$Re_B = \omega_0 r_0 (1 - \bar{v}_{r_0}) h_B/\nu$	is the Reynolds number for flow over blade;
$Re_T = v l/\nu$	is the Reynolds number for flow over measurement body;
$Re_c = n d_M^2/\nu$	is the centrifugal Reynolds number.

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