RESIDENCE TIME DISTRIBUTION OF SOLID PARTICLES IN AGITATED VESSELS EQUPPED WITH DEFLECTION BAFFLES

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An analog simulation and approximate theoretical analysis have been carried out for nonsteady state transfer of the solid phase along the height of continuous flow vessels equpped with agitators and deflection baffles. These were based on the diffusional model. It was demonstrated that at $0.8 > Pe_m > 0$ and when the stream flow velocity through the vessel is low compared to the settling (rising) velocity of the solid particles, the solid phase residence time distribution is practically equal to that in a perfectly mixed vessel. However, the average residence time of the solid phase is not equal to the average residence time of the slurry and slurry flow rate. Design equations have been derived relating the average residence time of solid particles in the vessel to mixing intensity. These have been experimentally checked in vessels with various types of mixers.

A number of articles [1-4] have been published on the distribution of solid particles in batch agitated vessels. However, this data is not adequate for mixing calculations in continuous flow apparatus. Actual concentration distribution of particles along the height of the vessel can be significantly different from the ideal distribution [5] which is represented by uniform solid phase distribution throughout the vessel volume. Due to this fact design of continuous processes taking place in solid–liquid systems cannot be carried out using ideal mixing equations which are applicable for process calculations in homogenous media. This is confirmed by the experimental results of [6] which demonstrate that data are required regarding the actual residence time distribution of solid particles in the vessel as a function of mixing intensity to calculate the process of agitated dissolving. This data is also required for establishing the dynamic characteristics of slurry mixing equipment in the design of control systems.

This article presents the results of investigation of the residence time distribution of solid particles in vessels equipped with agitators and deflection baffles.

Considering the relationship between the residence time distribution function and the dynamic characteristics of the vessel [7], the problem can be reduced to finding the response function to a step or impulse disturbance at the slurry inlet, i.e., nonsteady state solid phase transfer with steady state agitation.* According to the diffusional model of turbulent transfer,† transport of particles along the height of a continuous flow apparatus with constant concentration in the radial direction (this condition is satisfied during mixing in equipment with deflection baffles [5]) is described by the equation

$$\frac{\partial^2 x}{\partial z^2} - \mathrm{Pe_M}\frac{\partial x}{\partial z} + \mathrm{Pe}\frac{\partial x}{\partial \bar{\tau}} = 0. \quad (1)$$

* It is assumed that the change in solid phase concentration does not significantly affect the nature of liquid flow in the vessel.

† The applicability of this model to transport processes of suspended particles in agitated equipment is confirmed by [5, 8].

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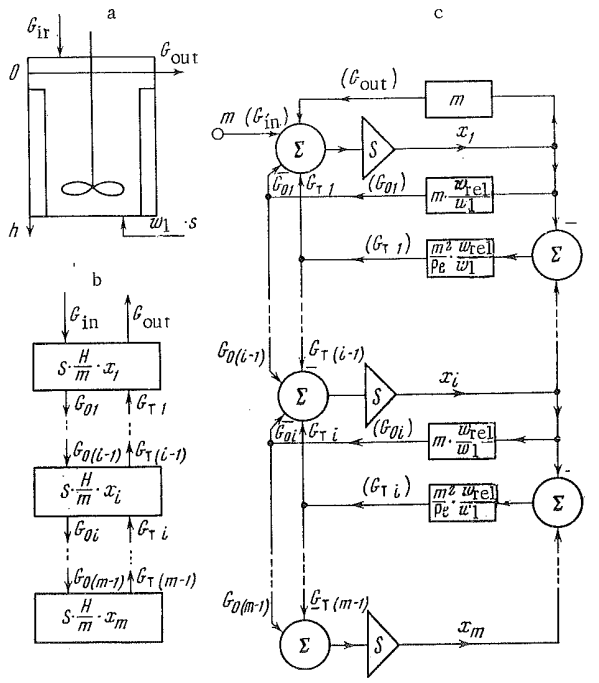

Fig. 1

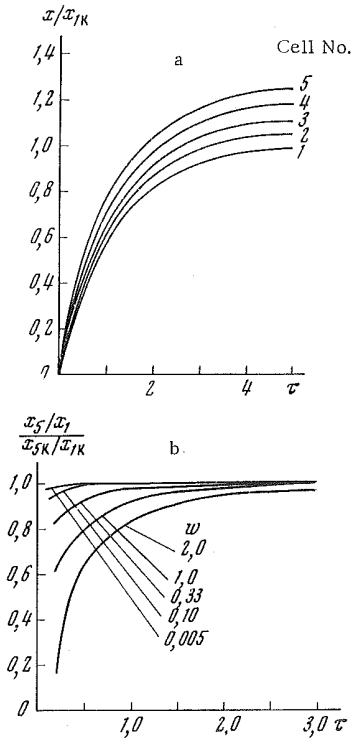

Fig. 2

Fig. 1. Model of solid particle transport with agitation.

Fig. 2. Results of modeling on an analog computer when $Pe_M = 0.3$: a) variation of concentration along vessel height at the time when w = 0.015; b) variation of relationship of concentration in the upper and lower cells at the times when w takes the values shown on the curves.

For step changes in inlet concentration the conditions for the same direction of change (increase in x_0 with higher x_{in}) are expressed as follows:

$$x = x_{ob} \quad \text{at} \quad \bar{\tau} = 0, \quad z = 0, \tag{2a}$$

$$\frac{\partial x}{\partial z} - (Pe - Pe_M)x_0 = 0 \quad \text{at} \quad \bar{\tau} > 0, \quad z = 0, \tag{2b}$$

$$\frac{\partial x}{\partial z} - Pe\, x_{oaft} - Pe_M x = 0 \quad \text{at} \quad \bar{\tau} > 0, \quad z = 1. \tag{2c}$$

Exact analytical solution of these equations as well as Eq. (1) under conditions given in Eqs. (2a), (2b), and (2c) is quite difficult. It is therefore useful to find an approximate solution which is convenient for practical calculations.

For a step change in inlet slurry concentration from x_{ob} to x_{oaft} at time 0 the solid phase material balance of the vessel can be written in the form of the integral function

$$Hx_{av_b} + w_1 x_{oaft}\tau = Hx_{av} + \int_0^{\tau} w_1 x_0\, d\tau. \tag{3}$$

Transforming this expression into dimensionless form we get by differentiation

$$\frac{dx_{av}}{x_{oaft} - x_0} - d\bar{\tau} = 0. \tag{4}$$

Since

$$x_{av} = \int_0^1 x\, dz, \tag{5}$$

it is necessary to know the concentration distribution of the solid particles along the vessel height x = f(z) to solve Eq. (4).

An approximate mathematical model of the particle transfer process was developed to determine the nature of the solid phase concentration distribution with height and its variation with time in a nonsteady state regime. The vessel (Fig. 1a) was represented as a system of ideally mixed cells of equal volume stacked on top of each other with a limited amount of mixing between cells (Fig. 1b). The diffusion equation was used to describe the transfer of particles in the direction opposite to their movement under the force of gravity. The concentration gradient was assumed to be equal to the ratio of concentration differences in adjacent cells to their height. The process of nonsteady state transfer of particles in the system of m cells with liquid inlet into the m-th cell, solids feed into the first cell, and slurry discharge from the first cell is described by the accumulation equations

$$\int \frac{H}{m} x_1 = \int (G_{in} - G_{01} + G_{\tau 1} - G_{out}) d\tau, \qquad (6,1)$$

. . . . . .

$$\int \frac{H}{m} x_i = \int (G_{0(i-1)} - G_{0i} - G_{\tau(i-1)} + G_{ti}) d\tau, \qquad (6,i)$$

. . . . . .

$$\int \frac{H}{m} x_m = \int (G_{0(m-1)} - G_{\tau(m-1)}) d\tau \qquad (6,m)$$

and stream equations

$$G_{0i} = x_i (w_{sett} - w_l) S,$$

$$G_{ti} = \frac{D_\tau (x_{i+1} - x_i) Sm}{H},$$

$$G_{in} = x_{oaft} w_l S,$$

$$G_{out} = x_1 w_l S$$

or in dimensionless form

$$\frac{dx_1}{d\bar{\tau}} = \frac{m^2 w}{Pe_M}(x_2 - x_1) + m(x_{oaft} - x_1) - m w x_1, \qquad (7,1)$$

. . . . . .

$$\frac{dx_i}{d\bar{\tau}} = m w (x_{i-1} - x_i) + \frac{m^2}{Pe} w (x_{i+1} - 2x_i + x_{i-1}), \qquad (7,i)$$

$$\frac{dx_m}{d\bar{\tau}} = m w x_{m-1} - \frac{m^2}{Pe} w (x_m - x_{m-1}). \qquad (7,m)$$

The concentration in the first cell x_1 is equal to slurry concentration at the outlet x_0. The block diagram of the model is shown in Fig. 1c. This model permits determination of the dynamic characteristic of the apparatus along different channels. A similar model was constructed for the case of solid and liquid phase inlet into the m-th cell and slurry outlet from the first cell. The model was programmed on a MNB-1 analog computer for m = 5. The calculations were carried out for step increases and decreases in slurry feed concentration. Pe_M was varied from 0.02 to 1.0 and w from 0.015 to 2.0. Curves of concentration variation with time in the cells are drawn in Fig. 2a. Analysis of the simulation results shows (Fig. 2b) that at $w \leq 0.1$ and $Pe_M < 1$ the slurry concentration distribution along the vessel height throughout the transition period is almost identical to distribution in the steady state. In practice the liquid stream velocity is normally low compared to the settling velocity of the solid particles and Pe_M does not exceed 0.6 (at higher values of Pe_M particles can settle to the bottom of the apparatus). Therefore, for an approximate analysis slurry concentration distribution function along the height can be assumed to be the same as the steady state distribution.

The following equation results from the material balance of the solid phase in the steady state

$$\frac{dx}{dz} - Pe_M x - Pe\, x_0 = 0. \qquad (8)$$

Solving this equation with the boundary conditions $x = x_0$ at $z = 0$ we get an equation relating the concentration distribution along the height to the solids concentration at the outlet of the vessel and the conditions of agitation

TABLE 1. Data on the Vessels, Agitators, and Slurries Used

Type of agitator	Vessel volume, liters	Agitator diameter, m	Settling (rising) velocity of particles, m/sec
Open turbine	1,7; 20; 50	0,05; 0,075; 0,1; 0,125	0,01; 0,018; 0,027; 0,055
Closed turbine	1,7	0,05	0,018; 0,027
Vane	1,7; 20	0,05; 0,075; 0,1; 0,125; 0,15	0,01; 0,018; 0,027; 0,055
Propeller	1,7; 20	0,05; 0,125; 0,15	0,018; 0,027; 0,055

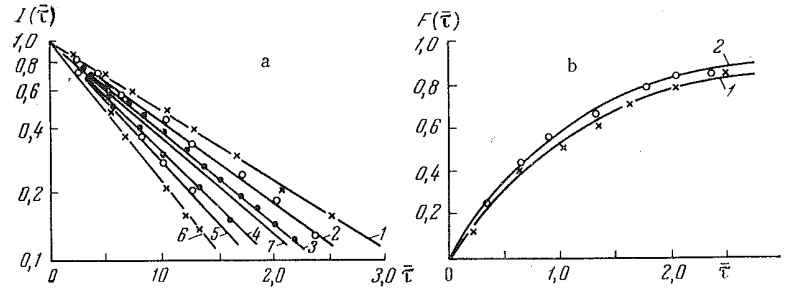

Fig. 3. Comparison of analytical results with experimental data: a) residence time distribution of solid particles in the vessel; 7) perfect mixing; b) comparison of experimental data with curves obtained on the analog computer.

Curve number	Type of agitator	D_{ves}, m	d_{mix}, m	n, rpm	w_{sett}, m/sec	Pe_M	Pe'_M
1	Open turbine	0,3	0,075	200	0,01	0,6	
2	Vane	0,3	0,1	550	0,027	0,3	
3	Open turbine	0,12	0,05	1250	0,027	0,12	
4	Open turbine	0,3	0,1	400	0,055		0,25
5	Propeller	0,3	0,125	450	0,055		0,45
6	Propeller	0,3	0,125	235	0,055		0,8

$$x = x_0[(1 + w)\exp(zPe_M - w)]. \tag{9}$$

Substituting the resulting expression into Eq. (5) we get, after integration

$$x_{av} = x_0\left[(1 + w)\frac{1}{Pe_M}(\exp Pe_M - 1) - w\right]. \tag{10}$$

Using expression (10) with the initial conditions $x = x_{ob}$ at $\tau = 0$ we get the solution of the nonsteady state transport Eq. (4) in the following form

$$\frac{x_{oaft} - x_0}{x_{oaft} - x_{ob}} = \exp -\frac{\bar{\tau}}{(1 + w)\frac{1}{Pe_M}(\exp Pe_M - 1) - w}. \tag{11}$$

It can be easily demonstrated that the value of $(\exp Pe_M - 1)/Pe_M$ is always greater than unity at positive values of Pe_M and approaches unity at $Pe_M = 0$. Taking this into account and limiting the area of applicability of the approximate solution to values of $w \leq 0.1$, to correspond to the results of the analog simulation, we simplify expression (11)

$$\frac{x_{oaft} - x_0}{x_{oaft} - x_{ob}} = \exp -\frac{\tau Pe_M}{\exp Pe_M - 1}. \tag{12}$$

Determining the value of x_0 from this and substituting it into Eq. (9) with $w \leq 0.1$ we get

$$x(z, \bar{\tau}) = \exp z Pe_M\left[x_{oaft} - (x_{ob} - x_{ob})\exp -\frac{\bar{\tau}\, Pe_M}{\exp Pe_M - 1}\right]. \tag{13}$$

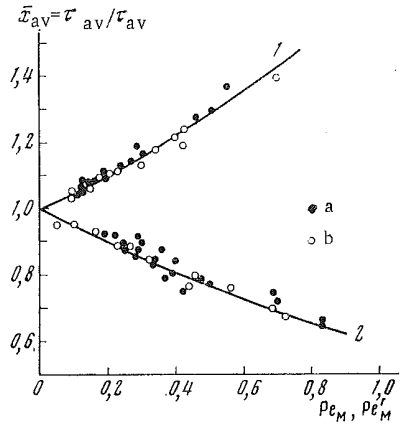

Fig. 4. Dependence of mean residence time of particles in vessel on agitation conditions: 1) $x_m = f(Pe_M)$; 2) $x_m = Pe'_M)$; a, b) experiments in dynamic and steady state conditions, respectively.

Equation (13) approximately relates the solid phase concentration distribution along the vessel height and its variation with time to the agitation conditions for a step change in inlet concentration. This expression approaches the exact solution of the equation of nonsteady state transfer of suspended particles (1) for conditions described by Eqs. (2) as $w \to 0$. Expression (12) is the desired residence time distribution function $I(\tau)$ of particles in the vessel. Considering the relationship between the integral function of residence time distribution and the response function of the system to a step disturbance [7], we get

$$F(\bar{\tau}) = \frac{x_0 - x_{ob}}{x_{oaft} - x_{av}} = 1 - \exp - \frac{\bar{\tau}\, Pe_M}{\exp Pe_M - 1}. \qquad (14)$$

Approximate equations (12) and (14) hold when the solid phase particles move, under the force of gravity, in the opposite direction to slurry flow in the vessel.

When the particle movement and flow directions are concurrent a similar analysis results in the equations

$$I(\bar{\tau}) = \exp - \bar{\tau}\, \frac{\exp Pe_M - 1}{Pe_M}; \qquad (15)$$

$$F(\bar{\tau}) = 1 - \exp - \bar{\tau}\, \frac{\exp Pe_M - 1}{Pe_M}. \qquad (16)$$

The results of the approximate calculation and the analog simulation were experimentally checked in the laboratory using standard agitators in cylindrical vessels 0.12, 0.3, and 0.4 m in diameter (Table 1). These were equipped with four deflector baffles which were 0.1D in width (D is vessel diameter).

Water slurries of aluminosilicate, ion-exchange resins, and polyethylene were used in the experiments. Properties varied as follows: particle density 900-2650 kg/m^3, average size 0.1-4 mm, and settling (rising) velocity 0.01-0.055 m/sec. Water and the solid particles were fed into the vessel separately: particles were fed into the top by a disc feeder while water was introduced into either the upper or lower part of the liquid layer from a head tank. The slurry was withdrawn from nozzles located either at the bottom or at the surface of the agitated liquid. Average liquid residence time was from 2 to 20 min and the steady state slurry concentration was 10-30 wt. %. Disturbances were introduced by step increases or decreases in solid phase feed.

Figure 3a shows the results of experimental determination of residence time distribution of particles of ion-exchange resin and polyethylene in a 20 liter capacity vessel with different types of agitators. Lines, calculated from Eqs. (12) and (15) are also shown in this figure. The value of the turbulent transfer coefficient D_T was determined from the following equation in this calculation

$$D_T = 0.12 n\, d_M^2\, Re_d^{0.09} (K_N \Gamma_d)^{0.36}, \qquad (17)$$

This equation was derived for vessels with deflector baffles in [5]. It can be seen from the figure that expressions (12) and (15) satisfactorily describe the residence time distribution function of particles in a vessel and the effect of mixing intensity on it. Figure 3b compares the experimental results with curves obtained by analog simulation. This comparison leads to the conclusion that the cell model with diffusional transport between cells can be used to simulate vessels for slurry mixing.

The results of the approximate analysis and experimental data indicate that within the limits of Pe_M and w investigated, the residence time distribution of solid particles in agitated vessels with deflection baffles corresponds to perfect mixing. However, the average residence time of particles $\tau_{T\,av}$ at modified Peclet numbers $\neq 0$, differs from the average residence time τ_{av} determined as ratio of vessel volume to slurry flow rate. Comparison of Eqs. (12) and (10) permits us to represent the distribution function for $w \leq 0.1$ by the expression

$$I(\bar{\tau}) = \exp(-\bar{\tau} / \bar{x}_{av}) \tag{18}$$

where $\bar{x}_{av}$ is the ratio of average slurry concentration in the vessel to the concentration at the vessel outlet under steady state conditions

$$\bar{x}_{av} = (\exp Pe_M - 1) / Pe_M. \tag{19}$$

When the direction of settling (rising) of the solid particles is concurrent with flow through the vessel

$$\bar{x}_{av} = \frac{Pe_M'}{\exp Pe_M' - 1}. \tag{20}$$

Figure 4 shows $\tau_{T\,av}/\tau_{av}$ calculated from experimental response curves for a step disturbance and measured by the intercept method under steady state conditions, as a function of agitation intensity defined by a modified Peclet number. The experimental values are located close to the curves calculated by Eqs. (19) and (20) and the discrepancy does not exceed the accuracy limits of practical calculations.

NOTATION

D_{ves} – vessel diameter, m;
D_T – turbulent transport coefficient, m^2/sec;
d_m – agitator diameter, m;
H – height to which the vessel is filled, m;
h – vertical coordinate, m;
N – power, W;
n – rps of the agitator;
S – horizontal cross section of the vessel, m^2;
w_l – liquid velocity, m/sec;
w_{sett} – particle settling (rising) velocity, m/sec;
x – solid phase concentration, m^3/m^3;
ν – kinematic viscosity, m^2/sec;
ρ – density, kg/m^3;
τ, τ_{av} – time, average residence time of the slurry in the vessel, sec;
$Re_{agit} = nd_M^2/\nu$ – Reynolds number;
$Pe = w_{sett}H/D_T$ – Peclet number;
$Pe_m = (w_{sett} - w_l)H/D_T$, $Pe_m' = (w_{sett} + w_l)H/D_t$ – modified Peclet number;
$K_N = N/\rho n^3 d_M^5$;
$G_d = D_{ves}/d_m$;
$w = w_l/(w_{sett} - w_l)$;
$z = h/H$;
$\bar{\tau} = \tau/\tau_{av}$.

Subscripts

b, aft – steady state before and after the disturbance;
av – average value;
o – vessel outlet.

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