Calculation of Power and Flow Capacity of Rotor / Stator Devices in VisiMix RSD Program.

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The program VisiMix RSD is related to the mixing devices that consist of rotating element – ROTOR- that is placed inside or outside a fixed cylindrical STATOR.
The main specific features of this type of equipment;

- small distance between the surfaces of rotor and stator
- high linear velocity of rotor surface
- presence of radial channels or openings in rotors and stators

It is accepted that rotation of the rotor creates centrifugal force. Due to this force, liquid media is sucked into central part of the device, is pumped through radial channels in rotors and stators. While passing the internal channels of the devise with high values of velocity gradient and shear rate, the media is subject to action of a relatively high shear stresses – much higher then occur in tanks with usual mixing devices.

The RSD devices are used in two ways. They can be installed inside a mixing tank, usually – along with other, low speed mixing device, or on by-pass pipeline. In the first case after the high shear treatment the 'treated' flow of media is injected radially into external volume around the RSD location and is distributed in the tank. In the second case the 'treated' flow is pumped to outlet part of the by-pass pipe and can be returned into any selected point of the tank volume.

Mathematical models and calculations in current VisiMix RSD version are related to the phenomena that take place in the channels of the RSD and are applicable to the both described RSD applications.

Flow of liquid in inner space of RSD can be presented as superposition of two flows – tangential flow in the space between rotor and stator and periodic radial flow through slits of rotor and stator. Accordingly, energy of motor is spent due to two effects – overcoming of hydraulic resistance to tangential liquid flow in cylindrical channel between rotor and stator ('shear' constituent of power P_{sh}) and supplying kinetic energy to the radial flow through the stator slots('flow' constituent of power P_{fl}):

$$P = P_{sh} + P_{fl} \qquad (1)$$

The following description is related to the most simple case of the RSD consisting of a central SLOT type rotor inside the SLOT type stator.

1. **Hydraulic resistance to tangential flow in cylindrical channel between rotor and stator .**

Power constituent P_{sh} is defined using the obvious relation

$$P_{sh} = M_{sh}*\omega \qquad (2)$$

where M_{sh} - moment created due to hydraulic resistance of surface of stator to tangential flow in the channel, N.m,
ω - angular velocity of rotor, rad/s.

This moment is defined as:

$$M_{sh} = \tau*F_{st}*R_{st} \qquad (3)$$

where τ - shear stress, N/m^2,
R_{st} - internal radius of stator, m.
F_{st} - internal surface of stator, m^2,

Shear stress is defined as:

$$\tau = C_f* \rho *V_{av}^2/2 \qquad (4)$$

where ρ - density of media, kg/m3,
V_{av} – average tangential velocity in the channel, m/s,
C_f - resistance factor.

Taking into account that difference of radiuses of rotor and stator is small as compared to radius of rotor, average value of tangential velocity in the cylindrical channel can be defined as:

$$V_{av} = V_{rot}/2 \qquad (5)$$

where $V_{rot} =\omega* R_{rot}$ – velocity of rotor, m/s,
R_{rot} – external radius of rotor, m.

Dependence of resistance factor on the flow conditions in turbulent and laminar regimes is described using the Two-K approximating functions (see W.B. Hooper ,The two-K method predicts head losses in pipe fittings, Chemical Engineering, Aug 1981,pp.96 -100):

$$C_f = K_1 / Re_{tg} + K_2 \qquad (6)$$

Here:

$$Re_{tg} = V_{rot}\,(2\,\Delta)\,\rho/\mu$$ – Reynolds number for flow in cylindrical channel,

For a simple design with internal rotor and external stator

$\Delta = R_{st} - R_{rot}$ – width of cylindrical channel, m,

μ – dynamic viscosity of liquid, Pa.s.

Parameters K_1 and K_2 are dependent on geometry of rotors and stators. Calculation of K_1 and K_2 for different RSD devices is based on experimental correlations connecting these values with main characteristics of rotors and stators, including number and sizes of slots and dimensions of cylindrical channels.

These experimental correlations have been developed using results of measurements of torque moments of RSD rotors at conditions excluding radial flow. Absence of radial flow was ensured by covering of external cylindrical surface of stators (tests with '**closed**' stator).

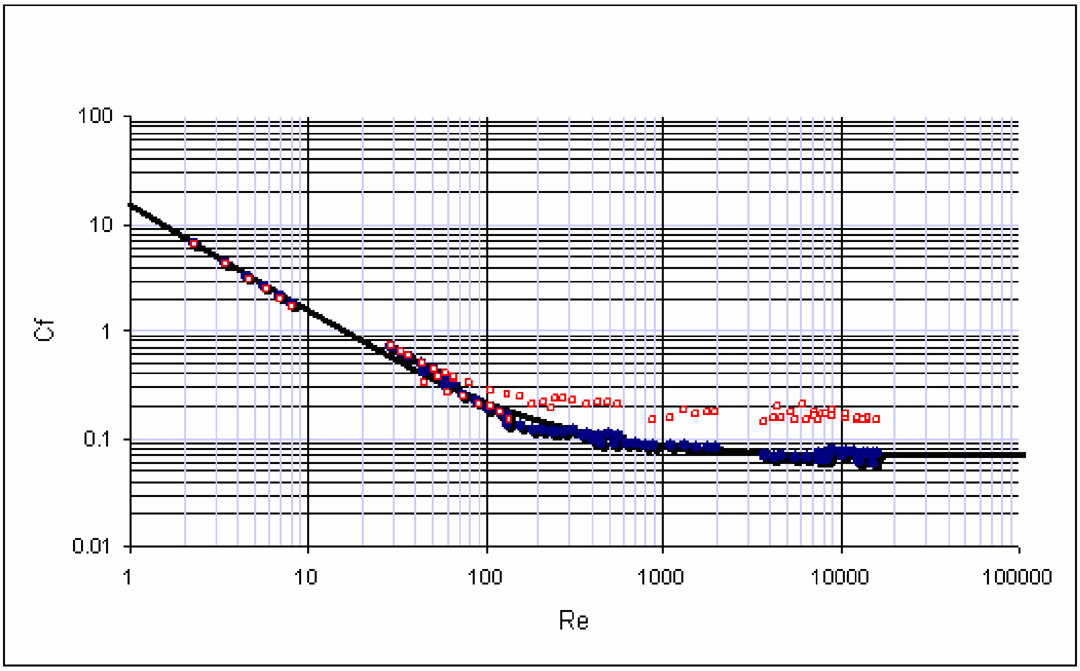

Figure 1. Experimental flow resistance functions. Black points and line – RSD with 'closed' stator slots, red points – open stator slots.

Measurements were performed with more then 30 combinations of rotor / stator dimensions. Ten Newtonian experimental liquids – water, glycerol and water - glycerol solutions with viscosity 1 – 960 cP were used. A typical experimental curve is shown in the Fig. 1.

Comparison of calculated values of torque moment created with tangential shear with the experimental values of momentum measured with 'closed' stator is shown in the Fig.2.

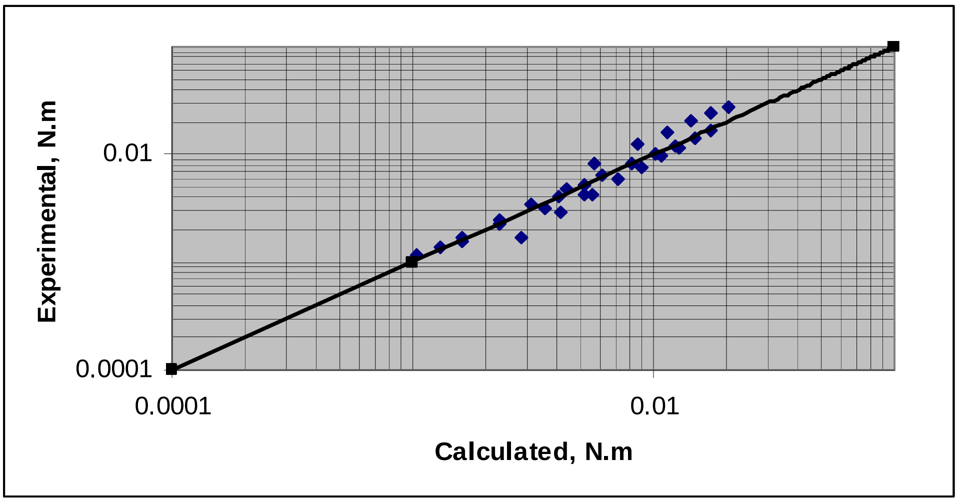

Figure 2. Torque moment of RSD with 'closed' stator. Comparison of calculated and experimental values.

For devices with a few rotors and stators the total 'shear' torque moment is defined as a sum of torque moments of rotors:

$$M_{sh} = \Sigma\ \tau * F_{st} * R_{st} \qquad (7)$$

2. Flow capacity and kinetic energy of radial flow.

Mathematical description of pumping capacity of RSD devices is also performed in terms of hydraulic resistance. It is based on assumption that due to acceleration in the rotor slots tangential velocity of liquid on the outlet of rotor is equal or very close to tangential velocity of the rotor defined above as:

$$V_{rot} = \omega * R_{rot} \qquad (8)$$

Accordingly to it, power spent for acceleration of the radial flow is defined as:

$$Dlt\_Pow = P_{kin} = Q\ \rho\ V_{rot}^{2} /2 \qquad (9)$$

Where Q – circulation flow rate created by RSD, m^3/s,
Dlt_Pow – difference of power of RSD with 'open' and 'closed' stator slots, W.

Calculation of radial (circulation) flow rate as a function of characteristics of RSD and media is based on obvious equilibrium of centrifugal pressure created by rotor and hydraulic resistance of the rotor and stator slots. Considering the RSD as a 'black box' and taking into account only average characteristics, the pressure / resistance equilibrium is described as:

$$P_{cf} = \Delta P_{res} \qquad (10)$$

where P_{cf} - centrifugal pressure created by rotor:

$$P_{cf} = \rho \int_{R_{in}}^{R_{out}} \omega^2 r dr \qquad (11)$$

ΔP_{res} - hydraulic resistance of rotor and stator slots to radial flow:

$$\Delta P_{res} = Cf_{fl}\, \rho\, W_{rot}^{\ 2}/2 + Cf_{fl}\, \rho\, W_{st}^{\ 2}/2 \qquad (12)$$

and Cf_{fl} - factor of hydraulic resistance for flow of liquid through the radial slots:

$$Cf_{fl} = K_3 / Re_{fl} + K_4 \qquad (13)$$

Reynolds number for slots is defined as:

$$Re_{fl} = \rho\, W_{sl}\, (2S_{sl})/\mu \qquad (14)$$

Where W_{sl} - average radial velocity in slot, m/s,
S_{sl} - width of slot, m.

W_{rot} and W_{st} are average (by time) velocity values of radial flow in slots of rotor and stator, m/s:

$$W_{rot} = Q /(Z_{rot} S_{rot}\, H_{rot}) \qquad (15)$$

$$W_{st} = Q /(Z_{st} S_{st} H_{st}) \qquad (16)$$

Parameters K_3 and K_4 in equation of hydraulic resistance (13) are functions of geometry of rotors and stators. They are calculated using empirical correlations based on results of experimental research.

For devices with a few rotors and stators Equations (11) and (12) are transformed as follows:

R_{out}

$$P_{cf} = \Sigma \rho \int_{R_{in}} \omega^2 r dr \qquad (17)$$

and

$$\Delta P_{res} = \Sigma Cf_{sl} \rho W_{rot}^2/2 + \Sigma Cf_{sl} \rho W_{st}^2/2 \qquad (18)$$

The equations presented above serve a base for calculation of the main operational parameters of RSD devices – power, flow capacity, shear rates, etc., and also of some characteristics of mixing in tanks with RSD devices. Comparison of calculated values of power and flow capacity for different RSD's with results of measurements in Newtonian liquids of different viscosity is shown in Figs. 3 – 6.

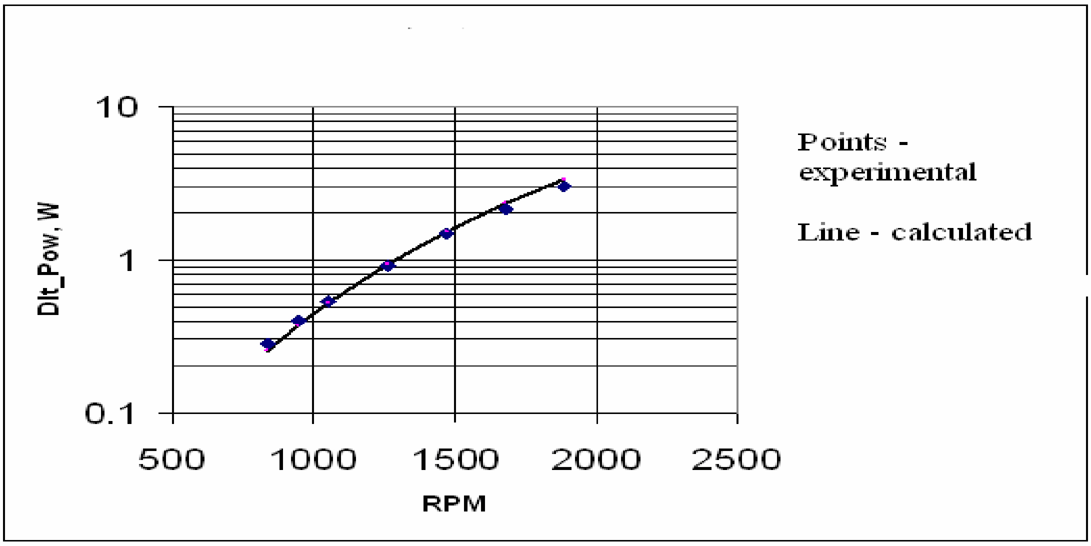

Figure 3. Power consumption for circulation flow. Comparison of calculated values with results of measurements. RSD data: stator external diameter 50 mm, rotor – 42 mm.

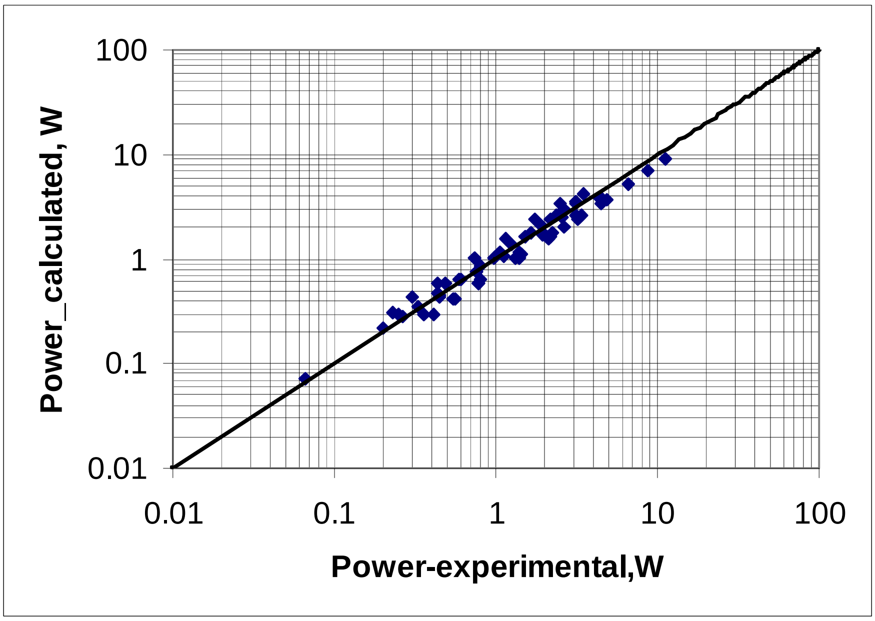

Figure 4. Power consumption of RSD. Comparison of calculated values with results of measurements.

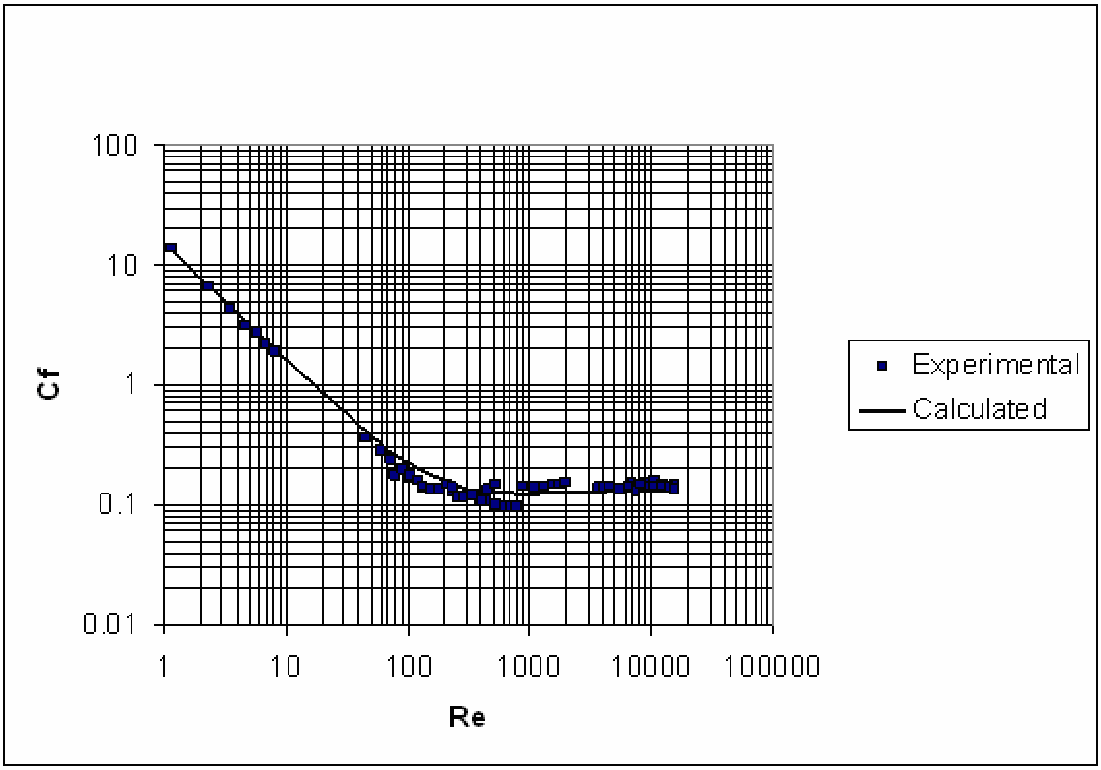

Figure 5. Flow resistance factor for RSD with 'open' stator. RSD data: stator external diameter 50 mm, rotor – 40 mm.

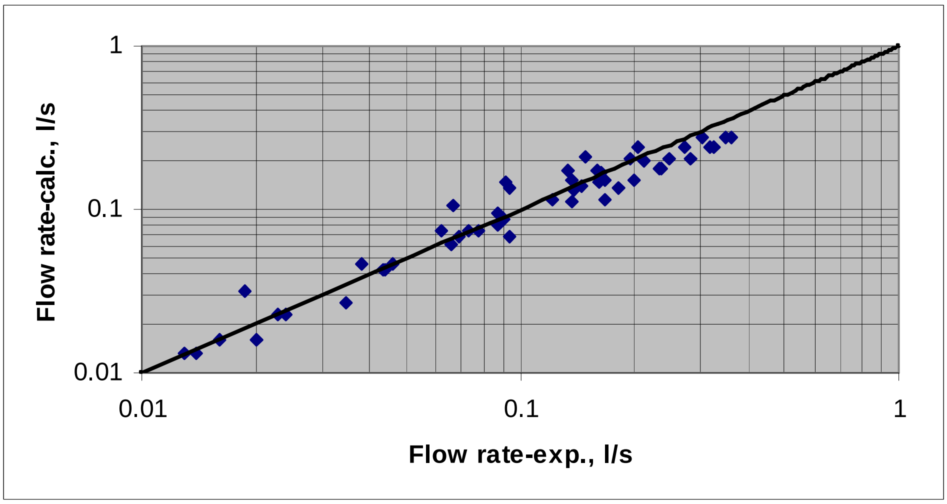

Figure 6. Flow capacity of RSD. Comparison of calculated values and results of measurements. RSD data: stator external diameter 50 mm, rotor – 42 mm.

3. Power and flow capacity of RSD in non-Newtonian liquids.

The method of calculation of power for RSD in non-Newtonian liquid is based on the connection between 'effective' viscosity and hydraulic resistance to tangential flow in the channels of RSD device. The physical background of the method is described earlier (see "A Method for Calculation of Effective Viscosity and Mixing Power in Non-Newtonian Media." by L.N.Braginsky. Annual Meeting of AIChE, Dallas, 1999, and " Generalized Method to Calculate Power Consumption when Mixing Viscous Newtonian and Non-Newtonian Media". by V.I.Begatchev, L.N.Braginsky and co-authors -Theor. Found. of Chem. Engng (USSR),USA, 1980, v.14, N1.).

The effective viscosity of non-Newtonian liquid at given flow conditions is understood as viscosity of such Newtonian liquid that shows in these conditions the same flow resistance. For cylindrical channel inside the RSD the flow resistance is described, accordingly to equations (4) and (6) as:

$$\tau = (K_1 / Re_{tg} + K_2) \quad * \rho * V_{av}^2/2 \qquad (19)$$

Where

$$Re_{tg} = V_{rot} *(2* \Delta)*\rho/\mu_{eff}$$

On the other hand, the shear stress can be presented as a function of effective viscosity and shear rate as:

$$\tau = \gamma_{eff} * \mu_{eff} \qquad (20)$$

Obviously, the shear rate γ_{eff} in this equation is some effective shear rate that is connected with the effective viscosity by rheological function:

$$\mu_{eff} = Func(\gamma_{eff}) \qquad (21)$$

The program handles two types of rheological functions:

Yield & Power model:

$$\tau = \tau_0 + K^* \gamma^n \qquad (22)$$

and Carreau model:

$$(\mu_{eff} - \mu_{min}) / (\mu_{max} - \mu_{min}) = (1-(\lambda \gamma)^2)^{n/2} \qquad (23)$$

Effective shear rate γ_{eff} and the corresponding effective viscosity for the given flow conditions are defined with a numerical solution of the equations (19)-(23).

In the course of experimental research, evaluation of γ_{eff} and μ_{eff} was performed accordingly to the a scheme presented in the Fig.7. Measurements were performed with 0.5% -5% CMC-water solutions.

Torque moment of RSD- measurement with closed stator .

Shear stress – based on experimental Torque moment .

Effective shear rate – value of shear rate that corresponds to the experimental Shear stress and to rheological function of the non-Newtonian media

Figure 7. Evaluation of shear rate in non-Newtonian media based on measurements of torque moments.

In the Fig.8 values of shear rates calculated using the described method are compared with values based on experimental measurements of torque moment in 5% CMC –water solution.

Comparison of power consumption of RSD's with 'closed' stators and results of measurements in different non-Newtonian liquids is shown in the Figure 9.

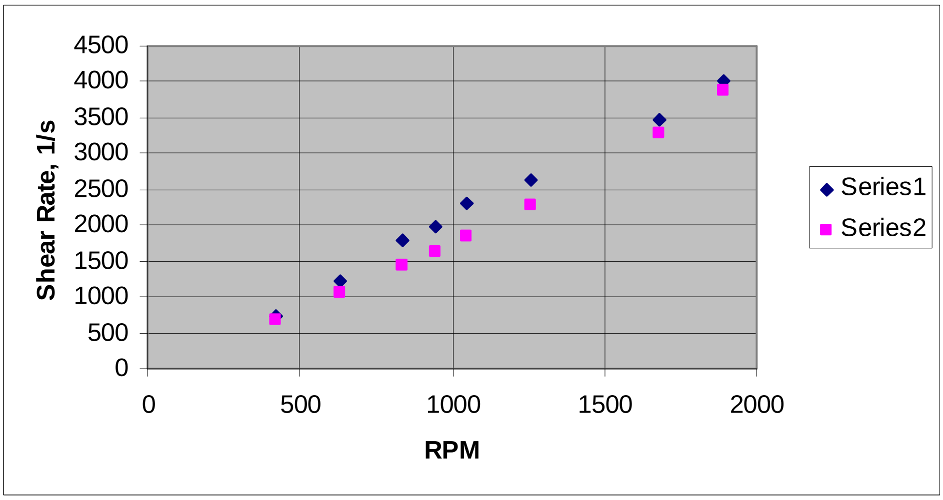

Figure 8. Comparison of experimental shear rates (blue points) with calculated values (red points). Media – 5% CMC –water solution.

Figure 10. Power consumption of RSD with 'open ' stator in non-Newtonian liquid.

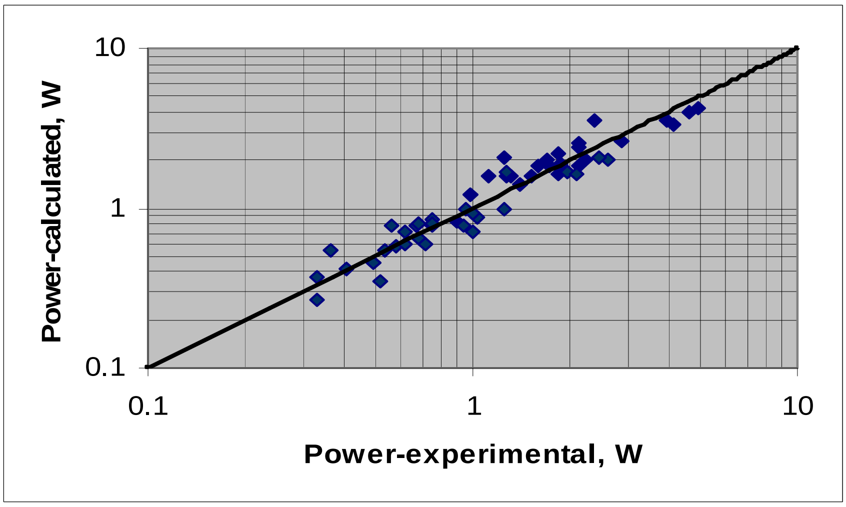

Figure 9. Non-Newtonian liquids. Power consumption of RSD with 'closed' stator.

As follows from equations presented above, values of effective shear rate and corresponding values of effective viscosity are connected with flow resistance in channels and must be considered as local values on the channel walls, i.e. in a thin hydrodynamic boundary layers on surfaces of rotor and stator. On the other hand, treatment of media that takes place in the whole volume of RSD inner space and flow resistance of RSD to radial flow depends on some average values of shear stress and the corresponding effective viscosity. These average parameters are assumed to be connected it specific power in the RSD channels:

$$\gamma_{aver} = \sqrt{(\varepsilon * \rho / \mu_{aver})} \qquad (24)$$

Values μ_{aver} defined with equation (24) are used for calculation of flow capacity and the corresponding constituent of power consumption of RSD (see paragraph 2 above) in non-Newtonian liquids .

Comparison of calculated power values for the ‘open’ RSD, based on this approximation, with results of measurements is presented in the figure 11